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The Shape of Cities: Geometry, Morphology, Complexity and Form

Why have cities not, long since, been identified, understood and treated as problems of organized complexity? If the people concerned with the life sciences were able to identify their difficult problems as problems of organized complexity, why have people professionally concerned with cities not identified the *kind* of problems they had? (Jacobs, 1961, p. 434.)

1.1 Understanding Cities

When Jane Jacobs posed her prescient question over 30 years ago, our understanding of cities was still dominated by the search for a visual order. As our immediate knowledge of the city is visual, it is perhaps explicable that urban problems which manifest themselves in cities are first associated with the destruction of visual order and harmony. The clear consequence of this has been the quest to solve, or at least alleviate, these problems by reimposing this order or developing it anew through city planning and design. Indeed, modern city planning still takes its inspiration from works such as Camillo Sitte's (1889, 1965) *City Planning, According to Artistic Principles*, which was published a little over 100 years ago. As long as man has sought to interpret the city, this has been mainly though the visual arts and architecture, culminating in the present century in the ideologies of the Garden City, the City Beautiful, and the Modern Movement. This deeply ingrained view of the city has had a profound influence on less artistic, more humanistic and somewhat more explicitly scientific approaches which in turn have sought to see the city through the need to assert statistical order in terms of homogeneity of its structure and the suppression of 'undesirable' diversity. Indeed, since Jane Jacobs elaborated her thesis, our understanding now augmented by the realization that the city presents a kaleidoscope of complexity, has hardly changed; planning and design still seek to impose a simplistic order on situations which defy our proper understanding and which we can only perceive as disordered.

Yet throughout history, there has always been an alternative view. From ancient times, towns and cities have been classified into those which grow 'naturally' or 'organically' and those which are 'artificial' or 'planned'. The distinction between these types is manyfold and often blurred, and of course there exists a continuum from organic to planned growth, many if not most towns being formed from elements of both. One of the key distinctions involves the speed at which cities change, while another relates to the scale of their development. Organically growing cities develop much more slowly than those which are planned. Cities which grow naturally are formed from a myriad of individual decisions at a much smaller scale than those which lead to planned growth which invariably embody the actions of somewhat larger agencies. Planned cities or their parts are usually more monumental, more focussed and more regular, reflecting the will of one upon the many or, at best, reflecting the will of the majority through their elected representatives. Finally, organic change involves both growth and decline, while planned change is more asymmetric, frequently embodying growth but rarely dealing with decline. Thus in this sense, a more complete picture of urban development is based on a backcloth of natural or organic growth interwoven both in space and time by planned development.

These distinctions articulate themselves in clear visual ways. Organically growing towns seem to fit their natural landscape more comfortably in that if decisions are smaller in scale, they reflect the properties of nature more closely as well as reflecting more intense concerns at the local level. The degree of overall control and coordination between such individual decisions is usually less explicit while the overall resources which govern such development are mobilized separately in their parts without regard to any economies of scale which might be generated centrally. The development which occurs is much less systematic and often irregular in form, and such irregularity of form conflicts with our intuition and predisposition to thinking in terms of the simplistic geometrical order based on the geometry of Euclid and the Greeks. Moreover, it is naturally growing cities which have led to the various biological analogies so popular in describing city growth since the work of Geddes (1915, 1949). Planned growth appears more man-made in that the patterns produced are more regular, reflecting more control over the natural landscape, and the mobilization and coordination of much larger quantities of resources devoted to the development in question. In history, such planned developments are invariably centered upon the areas of towns associated with political or religious power – palace and temple complexes, or with rapidly developing colonial towns, while in the modern age, retail and industrial developments in contrast to residential display some of the same regularity. However, it is impossible to identify solely organic or planned towns, for these two classes of development merge into one another in many different parts of the city and at many different scales.

In terms of the doctrine of visual and statistical order, organic towns when viewed in plan form resemble cell growth, weaving in and out of the landscape, closely following the terrain and other natural features, embodying the technology of movement through main transport routes, like spider webs or tree-like forms focussed on centers which usually contain the origin of growth. Their geometry seems irregular, although as we will be at pains to emphasize throughout this book, this should not imply 'disorder'. In contrast, planned towns display a geometry of straight lines and smooth curves, built on a directness of movement which can only be imposed from above, embodying some sense of man's direct control over nature through technology. Until this century, such planned developments were either parts of larger towns or very small complete towns, more at the scale of the village; although with the institutionalization of large scale urban planning in the last 80 years, much more grandiose plans for entire cities such as the British New Towns or capital cities such as Chandigargh and Brasilia have been attempted which embody a more perfect geometry. Nevertheless, most towns and cities provide a blend of both, usually containing elements of the planned within a backcloth of organic growth. One of the clearest and perhaps surprising examples is the Athens of the fifth century BC where the acropolis, the agora and straight streets such as the Panathenaic way were but isolated elements in a city whose "... corresponding architectural growth was ... slow and unsystematic and irregular" (Wycherley, 1962).

These differences between organic and planned growth strike at the very core of the way cities are developed and manifest themselves in every way we might conceive their study. In this context, we will concentrate on the geometry of cities, on their spatial properties as displayed mainly in two dimensions through their plans, and in this sense, we will emphasize their shape. Nevertheless, we are confident that our approach does not stand aside from the mainstream, but maps closely onto other ways of understanding cities through diverse disciplines within the arts, humanities, the social and the engineering sciences. Our starting point in this chapter will be the ways in which traditional and popular geometry has been fashioned to extend our understanding of cities. Wherever planned development has taken place, man has invoked the doctrine of visual order and imposed simple, regular geometrical forms or shapes on cities using the geometry of Euclid and drawing inspiration from such city building as far back as the Greeks. Yet during this century, and particularly since scholars, such as Jane Jacobs amongst others, have drawn our attention to the poverty of city planning in this ancient tradition, the paradigm of the visual order has come under intense scrutiny.

In parallel, the idea that naturally growing cities are in fact more workable, more efficient and more equitable, indeed more democratic, has gained credence as we have begun to probe the complexity which composes the way cities evolve and function. In the last 30 years, the gradual relaxation of the theoretical structures imposed on us through classical physics, mathematics and art which assume that whatever theory we develop must be simple, clear, workable and mechanistic, is leading to very new approaches to knowledge which appear more promising in the study of complex systems such as cities than anything hitherto. New approaches to time which embody discontinuity and to space which embody irregularity are becoming established and changing the philosophies to which we have traditionally ascribed. In this book, we will wholeheartedly embrace these new paradigms and demonstrate how we can begin to think of cities as systems of organized complexity whose geometry betrays a complexity of scale and form of which we have hitherto been largely unaware. To this end, we will suggest how urban theorists and city planners alike might move their world view a little closer to what we see as the 'true reality' of the ways cities develop and should be developed.

We will begin by tracing the various changing conceptions of how space and time have been abstracted across the broad sweep of human history. Our tendency to continually abstract through simplification manifests itself in the way we use mathematics to portray order and regularity, the way we conceive time as a continuous flow, and the way we perceive space as composed of simple geometries. But this is rapidly changing and we are entering a time when many of these traditional notions are being intensely scrutinized. In our quest to refocus the study of cities using these new ideas, we will first trace the juxtaposition of planned and organic urban forms and the ways these conceptions have dominated the study of urban form throughout history. From this review will emerge a deeper sense of how the morphology of cities should be understood in terms of their form and process, scale and shape, their statics and dynamics; and this will enable us to map out our approach which builds our understanding of urban form about the new geometry of the irregular – fractal geometry.

In essence the shift engendered by this approach is fundamental in that a theory of the fractal city breaks directly with the tradition that sees cities as simple, ordered structures, expressible by smooth lines and shapes which describe their overall morphology and the disposition of their elements. The change we seek to impress moves us closer to the view that cities are complex organisms, evolving and changing according to local rules and conditions which manifest more global order across many scales and times. In this, our view of cities is closer to modern biology than it is to either the visual arts or classical economics which have both influenced the study of cities and their planning so profoundly over the last century (Steadman, 1979). Nevertheless, our emphasis will still, in the first instance, be upon approaching the study of cities through their geometry and form, but always with this broader and deeper context in mind.

1.2 Ancient and Traditional Conceptions of Space

From the earliest examples of the written record, there is evidence that man has always made sense of the world through powerful simplifying abstractions which seek out the underlying principles and order in our experiences and perceptions. The power to abstract is one which probably sets man aside from the rest of the animal kingdom and it is clear that the ability to impress order and structure on diverse phenomena though casting aside detail irrelevant to the quest in hand, is strongly correlated with our conventional view of human progress. In short, abstraction leads to theory and theory enables the kernel of any phenomena to be isolated, defined and thence explained. From prehistory, such abstraction has been associated with the power to simplify the world visually and from the earliest cave paintings, man has sought to impose smooth geometry on art so that its meaning can be communicated in the simplest and most effective way.

Ten thousand years ago, the first towns developed when man moved from a nomadic existence to a society and economy based on more settled agriculture. This was what Toffler (1981) has called 'the first wave', beginning in the 'fertile crescent' centered upon the Rivers Euphrates and Tigris, in ancient Babylon. The evidence of man's attempts at visual abstraction and geometrical simplification of both natural and artificial phenomena come fast and furious from these times. Although this revolution was marked more by the natural or 'organic' growth of towns, there are many examples of 'planned' developments where man imposed his simple geometry on the land and upon the processes through which cities were sustained. The first cities show evidence of straight streets, of ordered land uses separated from one another, of vistas and monuments associated with the visual display of political and economic power in temple and palace complexes, of routes radiating from central places and of well-developed hierarchies of city systems consistent with elaborate agricultural and market economies. The earliest excavations have revealed urban agglomerations existing around 2500 BC; the Babylonian city of Ur, Harrapan cities along the Indus such as Mohenjo-daro and ancient Egyptian palaces as at Tel-el-Amarna all attest to the imposition of geometrically ordered streets and buildings following gridiron plans and focussed upon central points such as markets and temples (Morris, 1979).

In fact, there is no sense in the written record of any time when man's spatial sense of order was any less developed than in modern times, although the association of geometrical order with science and with the means to impose that order through technology has changed substantially since the first urban civilizations emerged. The Egyptians considered the world to be a flat plane yet the notion that the world might be round in some sense has been imbedded deep in our psyche since prehistory. The first known map of the world inscribed on a Sumerian clay tablet around 1500 BC, shows the familiar concentric and perhaps egocentric view of society, in that case centered in a circle about Babylon. This convention of centering or focussing social and economic activity in space around some powerful focus such as a city repeats itself throughout history when maps are made and plans proposed, and it has only been in the present century that there has been any sustained effort at thinking ourselves out of this traditional perspective.

It was the Greeks who first developed our visual senses to the point where art and science came to be treated as one, and where the imposition of geometry upon nature was first interpreted though the medium of science. It was the Greeks who first conceived of the earth as a sphere, and who first developed the requisite geometrical science to both demonstrate and use this understanding for the process of building cities. A long line of Greek scientists and geometers assembled a science and geometry which ultimately provided the foundation for the modern age and which essentially still dominates architecture and city planning to this day. The spherical model of the cosmos developed by Thales, Pythagoras, Herodotus amongst others and demonstrated using devastating measurement techniques by Eratosthenes, changed man's conception of space but more in matter of degree than kind. In fact, the notion that the earth might be a perfect sphere further impressed the idea that the 'true' geometry, the 'perfect' geometry, which was that which represented the highest form of art was that based upon the point, the line, the circle, the sphere and diverse combinations of the regular, in contrast to the irregular which remained beyond understanding (Berthon and Robinson, 1991).

During Greek and Roman times, the distinction between 'regular planned' and 'irregular organic' forms of urban settlement first appeared. In fact as we noted earlier, most towns grew organically as the product of many individual decisions made according to local rules and circumstances. But the Greeks and Romans left a legacy of planned towns, largely through their efforts at colonizing the known world, and it is there that the first examples of regular town plans based on the gridiron form make their appearance as at Miletus and Priene in present-day Asia Minor. The Roman military camp which could be assembled in a matter of hours also imposed geometrical order on places where none had been hitherto, and as technology developed to a larger scale, this geometry became imposed upon the wider landscape though long straight roads, walls and other man-made barriers as well as through large-scale agricultural cultivation. When the Roman world collapsed and Europe descended into her dark ages, what was left in terms of our knowledge and understanding of space was extensive and widely recorded in many treatises: Ptolemy's *Geography*, Vitruvius's *De Architectura*, and of course Euclid's magnificent exposition in his *Elements of Geometry* written some 300 BC, all of which rang down the ages to be rediscovered during Europe's Renaissance, precursor to the modern age.

For almost a thousand years from the division of the Roman Empire until the Crusades, the formal knowledge of geometry and science bequeathed upon us by the Greeks lay dormant in the monasteries or in the east in Constantinople where the crossroads with Islam gave it another twist through the development of algebra. In fact, the geometry was so deep-seated that it remained central to mainstream religious thought. There is a beautiful example of man's sense of the world and its geometry in the map produced by Isidore, the seventh century archbishop of Seville, which shows the world as round but formed as three continents, Asia, Europe and Africa, divided by the Mediterranean Sea, and the Rivers Nile and Don which we show in Figure 1.1. Isidore's map is more abstract than many before such as Ptolemy's, but it does reveal the extreme abstraction which has persisted until this present century in much map making, especially at the local scale. In the 13th and 14th centuries, Europe began to wake from its long sleep, trade revived, and the world view of society dominated for so long by religion came under increasing scrutiny. With this, the geometry and the science of the Greeks was rediscovered, literally reborn and almost immediately new advances were made in the development of geometry though the discovery of perspective. But it was in science that the real revolution in our perceptions of space came from, this time around.

Although the idea that the earth was a sphere had been known to the Greeks, the notion that the earth was center of the universe was central to religious belief, particularly to Christianity. However, the model of the universe based on interlocking spheres did not accord to observations of the motions of the planets and modern science from the 15th century generated increasingly precise observations of these motions. The great intuition, however, as to how these orbits fit together was made by Newton in the late 17th century and published in his *Principia* which established not only

Figure 1.1. A seventh century world geometry (from Berthon and Robinson, 1991).

the laws of gravitation which held for the observable solar system, but also the physical principles for diverse physical phenomena at many scales. Much of Newton's science and that of his contemporaries was deeply rooted in the notion of a perfect geometry. However, it was not the mechanics established by his insights, but the mathematics which he fashioned to present his science, which reinforced Euclid's view, still our conventional world view of geometry. In essence, Newton's mechanics depended upon the principle of continuity. Both space and time had to be continuous in the simplest possible sense for his theories to triumph. In short, the scale of physical systems and the forces which might change their scale could not admit any discontinuity which might change their form. Mass, for example, should be capable of being accelerated continuously, and if the force responsible were to cease, so would the acceleration and movement, but at a continually decreasing rate. This was the kind of science that embodied the principle of continuity, enshrined in the mathematics of the calculus which Newton and his contemporary Leibnitz invented to make all this possible. Such systems were said to change in a linear, continuous fashion both in terms of the space and scale they occupied and by which they were defined, and within the time frame of their existence.

During the 19th century, this type of physics based on the mechanics of Newton and the geometry of Euclid became the cornerstone of modern science. In other areas such as in biology, Darwin's theories of continuous evolution through survival of the fittest were also fashioned into the Newtonian mould, while the emergent social sciences began their quest to develop a science akin to physics based on reducing every phenomenon to continuously varying structures based on simple causal relations, embodying ideas of strong equilibrium and convergence. In short, by the end of the 19th century, the broad structure of science and associated knowledge was underpinned by concepts of pure geometry, the theory of continuous variation, the notion that all systems had some underlying simple set of forces, and the idea that their understanding could be pursued through successive reductionism. This was the 'majestic clockwork' as

Bronowski (1973) amongst others has referred to it. The world, however, was about to change, casting a doubt upon our age-old and perhaps superficial abilities to simplify through immediate and intuitive abstraction.

1.3 The New Science of Space and Time

At this point, it is worth posing a series of dichotomies which are not only useful in summarizing the changes to various world views which are relevant to our quest for a better understanding of space and time but are central to the changes in viewpoint which we will imply in the theory of fractal cities to be developed in this book. First we must contrast the notions of *simplicity* and *complexity*. Science stands at an edge between reality and mind, in perpetual tension between the need to simplify in order to understand and the need to provide a requisite variety in our theory to meet the perceived complexity. In one sense, however, the emphasis is more on simplicity, for great science, it is argued, seeks to provide the most parsimonious, hence the simplest and most elegant explanation, and success is thus judged through Occam's razor. In fact, we will argue that the science which is emerging everywhere in the late 20th century has found that previous standards of parsimony no longer admit the requisite explanation and thus we are now being forced to move to a higher threshold. In this sense then, our theories are becoming more complicated as well as dealing with new orders of complexity.

A second distinction is between *reductionism* and *holism*. Reductionist thinking has dominated physics and economics until quite recently, as indeed it has done biology, but there is a general and growing consensus that more holistic theory is needed which seeks to synthesize, not simply by aggregating the fine detail but by enabling the emergence of higher level form and function associated with new causes and forces. That 'the whole is more than the sum of the parts' may be a long-worn cliche of general systems theory, but ultimate explanations are no longer likely to be found in the quest for knowing more and more about less and less. To some extent, these issues smack of vitalism, and one small corner of our quest to counter this depends upon the logic of the ideas developed here. We have already mentioned our third distinction – the emphasis in Newtonian science upon the idea of continuity and the polarization of the *continuous* with the *discontinuous*. In essence, classical science has been entirely ineffective in coping with systems which display some abrupt change in behavior and in recent times, it would appear that more and more systems in very different domains manifest behavior patterns which cannot be treated using any kind of continuous formalism. In one sense, the idea that space is not continuous applies directly to cities, in that smooth change in physical form is clearly an abstraction when it comes to measuring and observing how the urban form evolves and shapes spatial organization. For a long time, science has been content to derive theory for idealized situations within the laboratory or within highly controlled situations, but increasingly, such science has been shown to be inapplicable to the real world, and continuity is one of the central problems inhibiting its applicability.

A fourth distinction involves the degree of homogeneity or heterogeneity which systems display, in essence the degree to which systems manifest *uniformity* or *diversity*. Systems which are intrinsically diverse and heterogeneous have for long been treated as being beyond science in some sense, while those systems for which the most pleasing explanations have been found are those which are well-behaved, controllable, homogeneous and ordered – that is, uniform in some sense. However, increasingly even the simplest systems betray a degree of complexity which departs from our traditional perceptions of uniformity and new theories are beginning to directly address the issue of explaining rather than suppressing diversity. A fifth dichotomy relates *certainty* to *uncertainty*. As we have begun to explain more and more, it seems that we are certain about less and less, that is, that our knowledge seems increasingly contingent upon time and space, upon the unique and the ephemeral. How is it, we ask, that the bounds of what we know seem to retreat a little faster than the rate at which we generate new ideas and insights? Is this progress? As we will argue throughout this book, this insight in itself is probably progress of a kind in a world of infinite variety and complexity, one whose nature we have only just begun to recognize.

Lastly, let us dwell briefly on the contrast between the *regular* and the *irregular*. In this book, we will be using this distinction in a very specific sense to draw out the differences between urban form conceived and perceived using the geometry of Euclid with that using the geometry of Mandelbrot (1983), the founder of fractal geometry, the geometry of the irregular. But the distinction is deeper and more far-reaching than this in that our penchant to abstract is strongly rooted in searching out the regular and dismissing the irregular. In short, we are predisposed to filter out that which we cannot cast into the geometry and the science of the regular although in doing so, we are in danger of casting out the very essence of what we need to explain. Science is only just beginning to grasp the notion that it is the irregular, the complex, the diverse, the uncertain, the whole system which is the proper domain of inquiry and to which we must reorient our quest.

In fact, at the end of the 19th century, classical physics was challenged, not by any of these opposites that are implied in the distinctions we have just sketched, but by the need to address basic forces in *relative* rather than *absolute* terms. Two different sources of anomaly emerged. First physical observations of phenomena involving the speed of light such as planetary orbits no longer seemed to fit Newtonian theory, while the conceptual problem of reconciling the space–time frameworks of observers light years apart loomed large. It was Einstein's intuition to visualize such problems and reconcile them by showing that the space–time continuum could no longer be treated as the absolute mould within which the universe existed if observers were to see the same thing at different positions in time and space. This represented the first loosening of a framework which had dominated scientific assumption since prehistory and as such represented the biggest challenge to man's intuitive grasp of the universe so far.

The second came close on its heels and involved not the very large but the very small. The continuing reductionism of physics took a major step forward in the late 19th century when the idea of the atom and its constituent parts became an intense focus of concern. As more and more particles came to be discovered – first the electron, then the proton, neutron and so on, a new framework for explaining the position of each was required, and with this came the startling conclusion that the actual position of such subatomic particles was uncertain. Because physical observations of position in time and space could only be made with physical forces, their actual position was fundamentally influenced by the parameters of the measuring device, and so was born the principle of uncertainty attributed to Heisenberg, a central postulate of the quantum theory. In fact, this notion of uncertainty was probably easier to accept in the social world where experience suggested that direct observation of phenomena often had an influence on the nature of that phenomena, and thus physicists were simply learning that the more remote the phenomena from direct observation, the more uncertain the outcome of that observation, a simple enough concept but one that again rocked long-held assumptions of the scientific world.

Reaction against reductionism too has been forced onto the agenda in many fields during the present century. In physics, there has been little success to date with the development of unified theory linking the very small to the very large, although there are intense efforts at the present time and there are signs of breakthrough. However, in less dramatic domains, particularly in the social and biological sciences, the idea that the whole system need be understood has become paramount. Aggregating micro to macro theory has proved to be virtually impossible in economics for example. Systems which contain many elements have required frameworks for their reconciliation which construct the whole from the parts and from the mid-century, the development of general systems theory has become significant. At first such theory was static in focus, intent upon explaining the form and function of systems at an instant of time, although in the last two decades such systems theory has been deeply enriched with new ideas concerning system dynamics and behavior. Moreover, the notion that systems might operate almost entirely using local forces which ultimately add up or aggregate to global order has also gained ground as it has become clearer that the very small and the very large can be different aspects of the same underlying system phenomena.

These changes in world view have had quite profound effects on our scientific approach to space. Throughout the 20th century, the idea of visualizing phenomena beyond the first three physical, and fourth, temporal, dimensions has become important. In many disciplines, the focus has been upon dimensions other than the four basic ones where space and time have been seen as simply the matrix within which more interesting and significant actions and forces exist, and this has been particularly the case in the social sciences. In economics for example, the predominant concern has been with the way various actors and agencies establish a competitive equilibrium through networks of markets and monetary allocation, such theories being largely independent of the space in which such systems exist and largely suppressing the temporal dynamics of such behavior in rigid assumptions concerning convergence and equilibrium. Anything which threatens to destroy the elegance of the equilibrium such as the imperfections posed by space and time have been ruled out of court. It is thus no surprise that economics has little or nothing to say about most current economic events which thrive upon such imperfections and although such theory is under intense scrutiny at present, it will take at least a generation for economics to reestablish its theoretical sights.

The same has been true for other social sciences, sociology and psychology for example where the space–time matrix has been simply assumed to be a given, neutral with respect to its effects upon the phenomena under study. This lack of a geometrical perspective in the social domain has been both liberating and constraining. It has meant in fact that social science has long avoided the trap of physical determinism but at the same time it has meant that the physical constraints of space and time have had little influence in social explanation where often such influence is important. In the study of cities for example, it has kept the social and artistic approaches separate except through the pragmatism of geography, and it has inhibited the development of a theory of city systems which is a relevant synthesis of social process and spatial form. In practice, this dichotomy can be seen at its most extreme when commentators and researchers dealing with the same subject using the same jargon present their ideas in diametric opposition through entirely visual or entirely verbal media. Urban theorists from the social domain have found the visual paradigm to be empty for their study of social process while those from the visual arts have found social processes to be impossible to relate to the manipulation of physical space which represents the long-standing medium for city planning and design.

Yet changes in our conceptions of space and time which see irregularity and discontinuity as reflecting a new underlying order and system do perhaps provide a fresh perspective as to the impact of physical determinants on social and economic processes. The emergence during the last 20 years of a mathematics and a geometry in which discontinuous change can be ordered in terms of catastrophes and bifurcations and where sudden change can be easily accounted for, has helped show the importance of formal dynamics to many fields. More recently, the development of theories of chaos in which deterministic systems generate behavior paths which are unique and never repeat themselves are finding enormous applicability in qualitative studies of system behavior and structure in the social and biological sciences. In fact, there are many physical systems such as the weather which are subject to the same underlying complexity and the notion of an intrinsic order based on strange attractors which can only be envisaged in the higher geometry of their mathematical space has become central to the study of many real systems. In biology too, the notion of smooth change or evolution has also been informed by these theories which explain the importance of punctuated equilibrium, sudden species development and ecological catastrophe. And in all of this, the smooth geometry associated with Euclid which has dominated our thinking for so long is giving way to a geometry of the irregular which is still ordered but where the order repeats itself across many scales and through many times and where such irregularity is clearly consistent with observations and measurements of our most interesting systems.

All of these changes in world view are tied up with the emerging science of complexity (Lewin, 1992), in turn being different facets of the kaleidoscope of complexity which science seeks to understand. When Jane Jacobs (1961) wrote about the need to understand cities as problems of organized complexity, she was invoking ideas from general systems theory, of a more speculative kind and associated with the writings of Warren Weaver (1967). Weaver argued, as we have done here, that science developed from the 17th to the 19th centuries dealing with problems of simplicity, two-body problems amenable to linear mathematics and strict determinism. The emergence of quantum theory shifted this balance to problems of disorganized complexity where the predominant characteristic method of explanation was statistical. But between, there lay many problems amenable to neither approach, "... problems which involve dealing simultaneously with a sizable number of factors which are interrelated into an organic whole" (Weaver, 1967). In short, such systems are those in which the emergence of organization is reflected in their form or morphology. We are now in a position to begin to develop some of these ideas in our study of cities, but before we do so, it is worth reviewing examples and principles of the geometry describing a range of city shapes beginning with the planned city, the city of the ideal, the city of pure geometry.

1.4 The City of Pure Geometry

If there has been any significant change in our visual sense of the city through history, this has been in the nature of the way it has been abstracted and represented. From a contemporary perspective, there appears to have been increasingly abstract representation of urban phenomena in visual terms as we delve further into the past which manifests itself in less realism and greater simplicity than is now acceptable. Currently, with more media to record than at any time in history through photographs, digital imagery and the like, city plans and maps from the past seem to abstract away too much while portraying some detail in almost surrealistic ways. Mapping, now perhaps, is also considerably more single-minded in purpose, and the sort of detail contained in historical maps relating to people and events as well as places suggest that the visual records of the past were for somewhat different and more comprehensive purposes than those we employ today.

An excellent example of this visual simplicity is contained in one of the earliest town plans known which represents the shape of an Assyrian military encampment and the segregation of its land uses some 2000 BC (Kostof, 1991). In Figure 1.2, we show this plan which depicts a circular and fortified town, divided by two axes into four quarters where the pictures in each symbolize the usage of these areas. The plan was embodied as a relief on the wall of a temple in Nimrud (in present-day Iraq) and as such is one in a long line of gridiron plans used for rapid development most obviously associated with military camps, but also widely used for colonization. This Assyrian example, in fact, shows all the elements which repeat themselves throughout history in terms of imposing and developing cities based on pure geometry: the circle which invariably encloses and bounds development as well as focussing upon the core, the straight streets and routes which form the structure of the grid, the blocks which represent the

Figure 1.2. The earliest depiction of the city of pure geometry (from Kostof, 1991).

interstices within the grid, the clear segregation of uses which is often imposed within such planned forms, and the fortified outer wall which was a feature of many cities until the present century when the technology of war went beyond this need.

The first evidence we have of highly ordered geometric forms is associated with either very rapid physical development, such as in military camps, or with more monumental, larger scale building related to the demonstration of political–religious–economic power within the city, such as in palace and temple complexes. For example, the camps used by workers to construct the pyramids and other monuments in ancient Egypt were laid out according to the strict principles of the grid, while colonial cities from the earliest Greek civilizations represented a more permanent but nevertheless rapid application of the same principles. Miletus and Priene are the archetypes, but there are many other examples which have been documented (Morris, 1979; Wycherley, 1962). The grid is also repeated in the development of larger complexes associated with the display of wealth and power throughout the ancient civilizations. All the important cities of the ancient world, Babylon, Knossos, Mycenae, Athens and of course, Rome, provide much evidence of a well-ordered geometry largely built around the gridiron as a basis for the construction of temples, market places, civic buildings and organized leisure in terms of sport and drama.

It is of interest that circular geometries are much less obvious and by association, much less used in city building up to the middle ages. Circular forms in a sense represent a natural bound for any city which is based on some central focus around which the major economic and political activity takes place. In this sense, most cities when examined in terms of their boundaries and edges, unless heavily constrained by physical features, are organized in some circular form, perhaps distorted along transport routes into a star shape, about some central point, usually the origin of growth. In fact, it is hard to find many examples where such circularity has been invoked as the basis of a geometrical order in towns before the medieval era. The Greeks did introduce radiality into their grids occasionally and there was some preoccupation with the use of the circle in theater complexes and stadia. The Romans did the same with their circuses and also in more detailed building through their invention and widespread use of the semi-circular arch. But it was not until the late middle ages and the Renaissance that cities really began to exploit the geometry of the pure circle. This perhaps was due to the lesser control over the geometry through the then available building technology although it is more likely that this may have been a purely aesthetic difference between ancient and modern, a difference in taste.

The best examples, of course, of the use of the grid come from the Roman military camp or *castra*, which is still the basis of many towns plans in contemporary Europe as evidenced best in England in towns ending with the word – *chester*. The main axes – the decumanus and the cardo – of such grids marked out the center of the camp where dwelt the legate, the legionary commander, and as the Assyrian map suggests arrayed around this were more specialized uses serving the legion, with the barracks banished to the edge of the camp often with recreation (the circus, amphitheater, etc.) beyond the wall. The Roman camp also marks the typical scale at which town plans were visualized and depicted up to these times. Although towns could be depicted in terms of their growth at a scale which abstracted away from the actual building and streets, this was very rare. The norm was to represent the town in terms of buildings and streets, and often to impose the geometry of the straight line on forms that clearly did not meet such geometrical purity in reality. However, the size of typical towns up until the modern age was so small and their form so compact that the sort of exploding metropolis reminiscent of the growth of London or the eastern seaboard of the United States which will be examples of our concern here, simply did not exist. This too goes some way in explaining their typical depiction.

The descent of Europe into its dark and middle ages led to the disappearance of the city of pure geometry. Towns looked inward; their form was compact although irregular and idiosyncratic, buildings huddled around the center which by now was church and market square. In fact, the notion of the circular city was much in evidence during these times to be fashioned a little later during Europe's Renaissance in more geometric form. In some instances, in the case of planned towns, for military purposes of control at borders, for example, the grid was still being used as it was wherever speed of development dictated its use, a fine example being the crusader port of Aigues-Mortes in the Rhone delta (Kostof, 1991). But what did develop quite distinctly during this period was a concern and fascination for elaborate fortifications based on regular but discontinuous geometries which maximized the amount of space available for the defense of a town.

A clear example of the succession of styles from Roman to medieval and beyond can be seen in the growth of the town of Regensburg on the southern bank of the River Danube which we illustrate in Figure 1.3 (Morris, 1979). In the year 350, the Roman settlement displays the clear grid of the

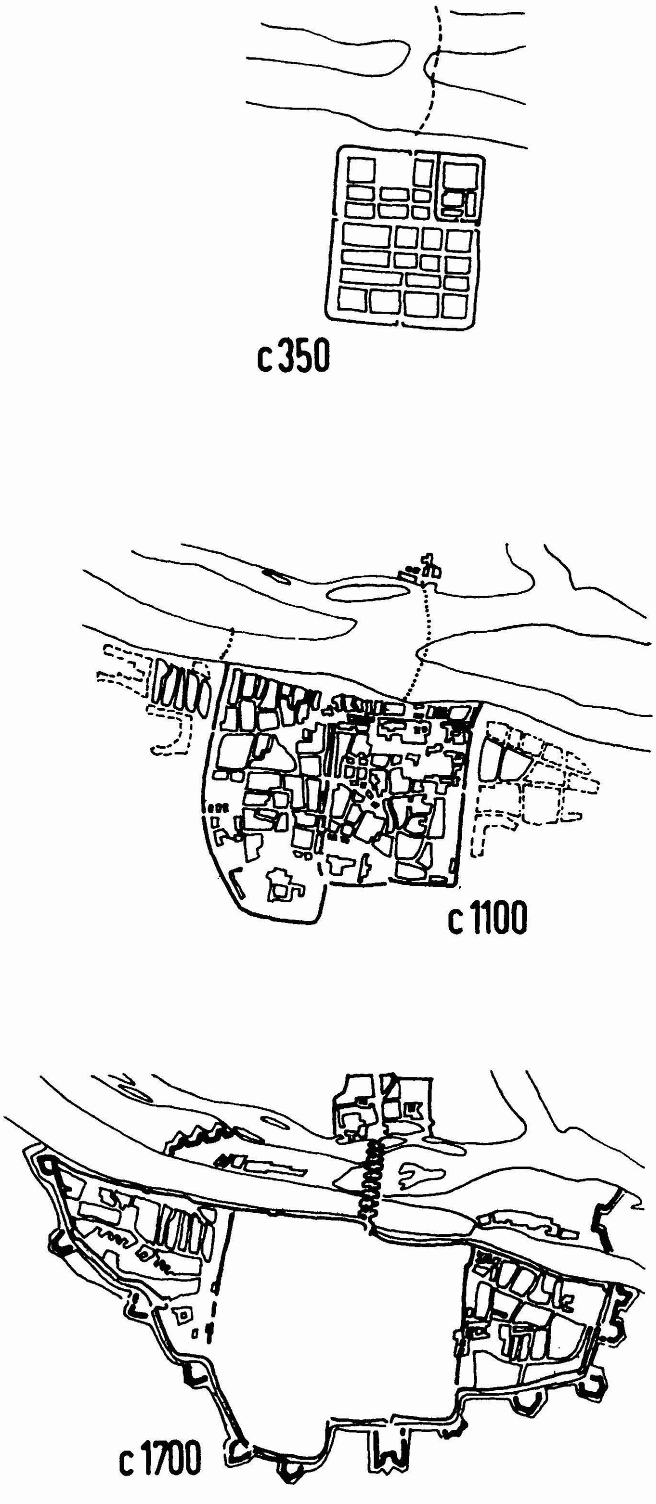


Figure 1.3. A succession of geometries: Regensberg from Roman times (from Morris, 1979).

original castra while by 1100, although this has collapsed into the huddle of medieval buildings, the effect of the original grid is still apparent. The fortifications which encircled the town by 1700 represented a tight bound on growth, although the focus of the town is clearer in its approximately circular expansion from the original center of settlement. The fortifications based on regularly spaced triangular displacements from the straight wall represent the classic style of fortifying towns to increase the wall space available for their defense and as such represent a kind of space-filling phenomenon which as we shall see in the next chapter, is reminiscent of the regular fractal called the Koch or snowflake curve.

The Renaissance, however, was the time of high theory for the city of pure geometry. The rediscovery of the architecture of Greece and Rome through the written works of scholars such as Vitruvius (Bacon, 1967) led to massive experimentation and speculation on ideal town forms. Combined with developments in the architecture of fortification, the discovery of perspective which generated the need for a radial focus in the plan as well as within the three-dimensional massing of the city, and the need for regularly laid out city blocks, ideal town plans were much more ambitious than anything previously and such was the strength of commitment and belief in the new order, that the ideal became real in many instances. Figure 1.4 shows two such ideals; the first in 1.4(a) is based on Vitruvius's (republished 1521) first book which is somewhat perplexing in that it established an ideal in the circular plan, something as we have remarked, that did not exist in Greek or Roman city building. The second plan in Figure 1.4(b) is that which was actually built for the city of Palma Nuova outside Venice usually accredited to the Italian architect Scamozzi (Morris, 1979).

Many similar ideal town geometries were suggested as we will note in the next chapter although perhaps the finest which was built is Naarden in Holland whose plan is as close to the original as any. In fact, many such ideas were incorporated into existing cities such as we see at Regensburg in Figure 1.3 as well as in much larger cities such as Paris, Rome and Vienna where idealized fortifications were continually under construction. Examples of more regular circular geometries also date from this time, one of the best examples being Karlsruhe which we picture in Figure 1.5 (Morris, 1979; Kostof, 1991). Nevertheless the circular town form was embodied much more thoroughly within existing towns in the form of foci for radial streets and the strategic disposition of circles and squares. Excellent examples date from the replanning of Rome under Pope Sixtus V in the late 16th century, Hausmann's Paris in the mid-19th century, Nash's Regent's Park in London, and l'Enfant's plan for Washington DC which was modified by Ellicott, the last two both being implemented during the early 19th century.

If the circle was to gain the ascendancy in Baroque Europe, it was the grid that complete dominated the development of American cities from the late 18th century onwards. Cities in the New World resembled those in the old until the early 19th century when rapid expansion led to widespread application of the gridiron as a matter largely of speed and convenience, and perhaps through a sense of modernity – a break with the past. New York or rather Manhattan island is the example *par excellence.* Town after town which was laid out in the western expansion of settlement in North

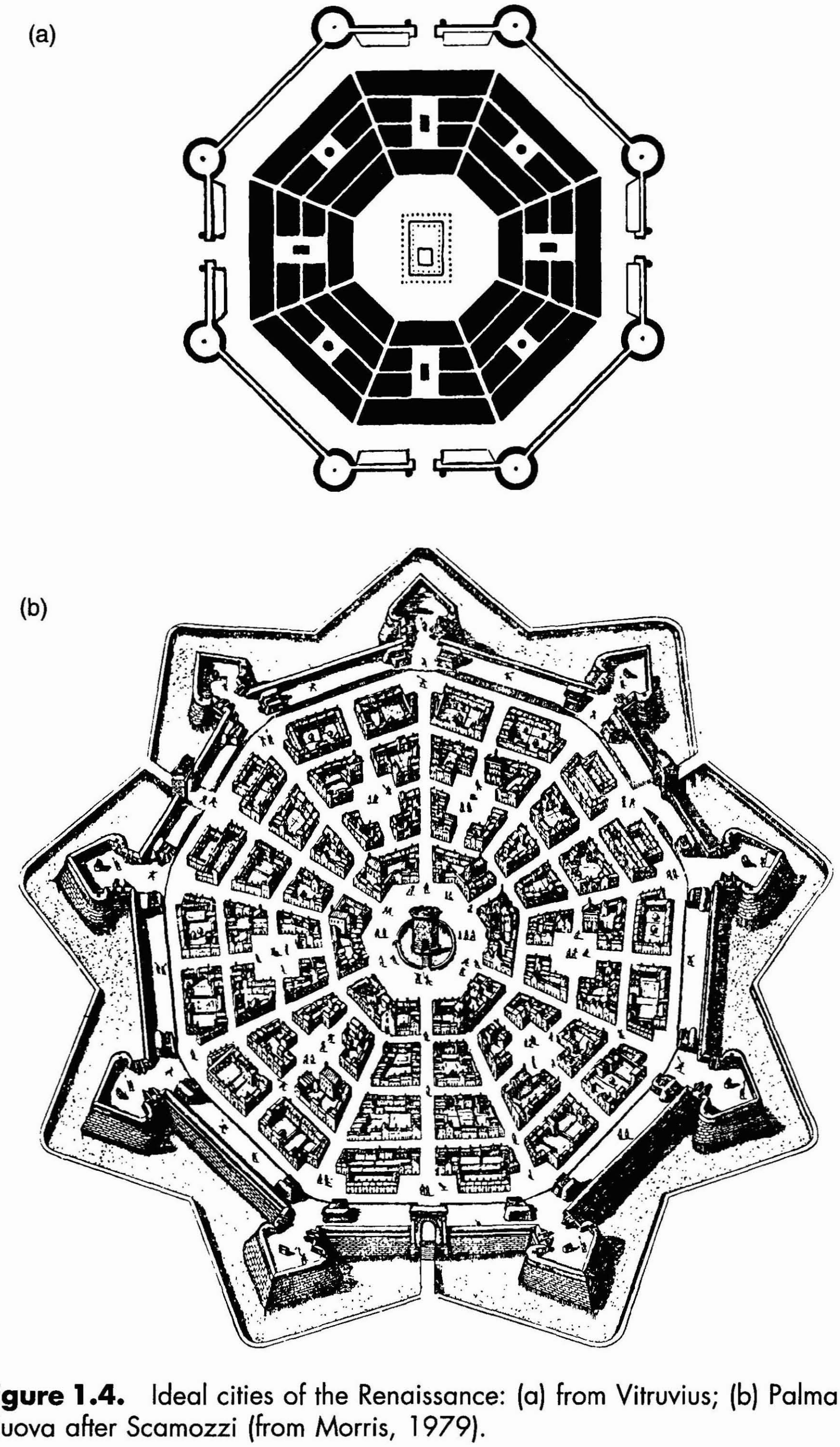


Figure 1.4. Ideal cities of the Renaissance: (a) from Vitruvius; (b) Palma Nuova after Scamozzi (from Morris, 1979).

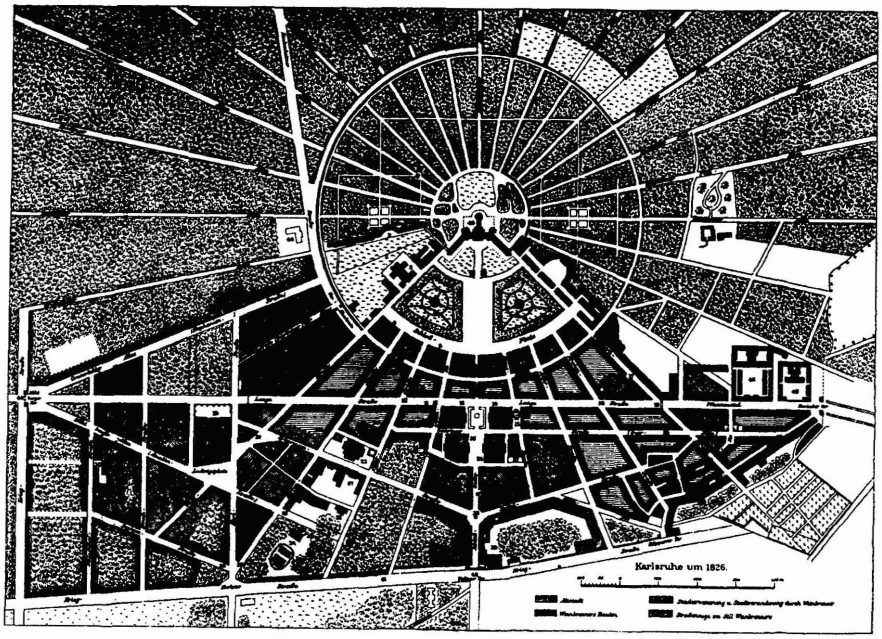


Figure 1.5. Circular towns: Karlsruhe (from Morris, 1979).

America conformed to the grid as Reps' (1965) *The Making of Urban America* so clearly demonstrates. Yet during these years too, there are few documented examples of how geometrically ordered plan forms actually developed; that is, concerning the extent to which such plans were modified. As we have reiterated throughout this chapter, such plans usually only exist for a snapshot in time and as such, once implemented do in fact begin to adapt to the physical and other constraints of settlement as well as to the actions of individuals working with a different purpose to that of larger agencies. Two examples of the extent to which pure geometry guided development, however, are worth illustrating. First, the town of Savannah, Georgia, was laid out in gridiron fashion in the 1730s by colonists from England and with surprising commitment given the rapid development during these years, the residents of Savannah grew their town according to the grid for the next 100 years. The evolution of the town is shown in Figure 1.6 and is one of the very few examples of urban growth clearly built on purely geometric principles.

The second example is more prosaic and it concerns the development of a circular town in southern Ohio named Circleville. Circular town forms as we have indicated are almost entirely absent from the New World although in the 1820s, such a form was adopted for this land on which had stood circular Indian mounds which may have influenced the shape of the plan and the naming of the town. However for diverse reasons, some clearly related to the use of space, the plan was 'redeveloped' some 20 years after it had first been laid out so that it might conform to the more standard grid. Remarkably, the agency responsible for carrying out this change was called the 'Circleville Squaring Company' and its actions are clearly recorded in the systematic transformation of the circular town plan into a

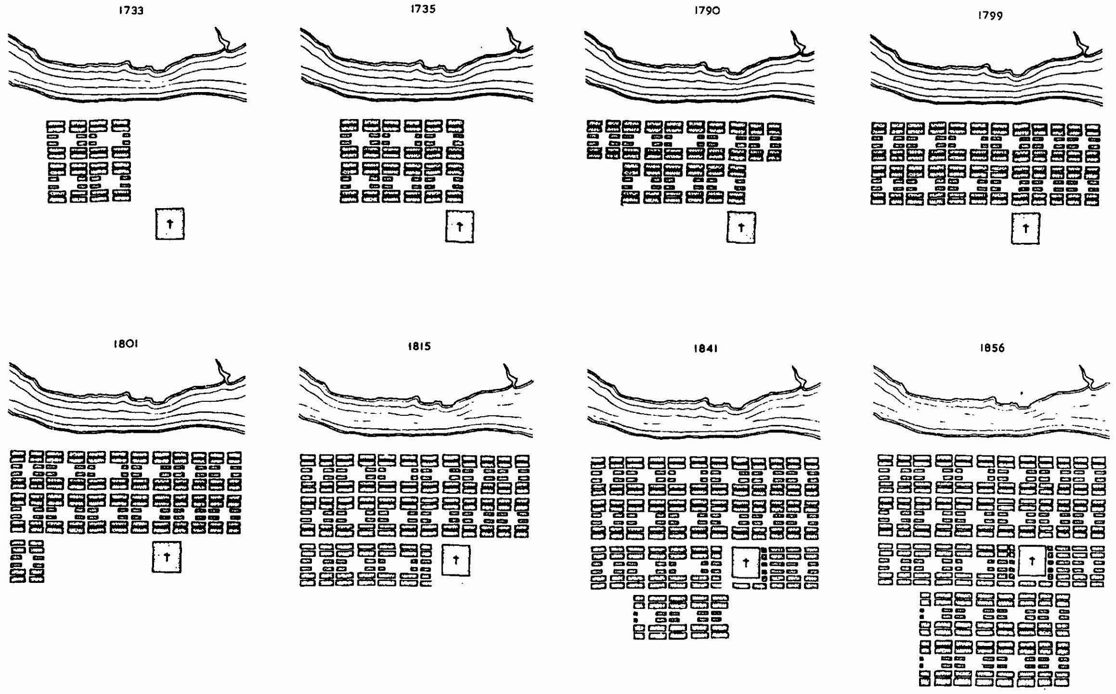


Figure 1.6. Regular cellular growth: Savannah 1733–1856 (from Reps, 1965).

grid as illustrated in Figure 1.7. Aside from the somewhat idiosyncratic example this poses, it does show the power of pure geometry in city building as well as reinforcing our popular and deeply-ingrained perception of what constitutes 'good design'.

During the present century, there has been a distinct shift to geometries which combine perfect circles and squares and the like with more sinuous although still smooth curves. There has also, in the last 50 years, been a major shift towards conceiving cities in terms of ideal network geometries based on communications routes, largely road systems. Architects and urban designers have exuded more confidence too in their quest to build the city of pure geometry, suggesting larger and larger idealizations of the old ideas. In the late 19th century, more abstract conceptions of the ideal city system based on social and economic ideas of utopia became important in movements such as the Garden Cities (Howard, 1898). These are so significant that we will deal with them in a later section for what they imply concerning urban form is pitched at a different spatial scale from the ideas of this and the next section. But we must point to the most significant of the 20th century physical utopias and we will begin with Le Corbusier.

The *Ville Radieuse* is perhaps the most important of Corbusier's statements about the future city and in essence, it is based not on any specific notion of grid or circle, but upon the idea that the city should exploit its third dimension much more effectively through tall blocks, thus releasing the ground space for recreation and leisure. In fact, Corbusier's ideas are best seen in his plan for the Indian capital of Chandigargh which is illustrated in Figure 1.8(a). The form in fact is one based on a grid, the scale of

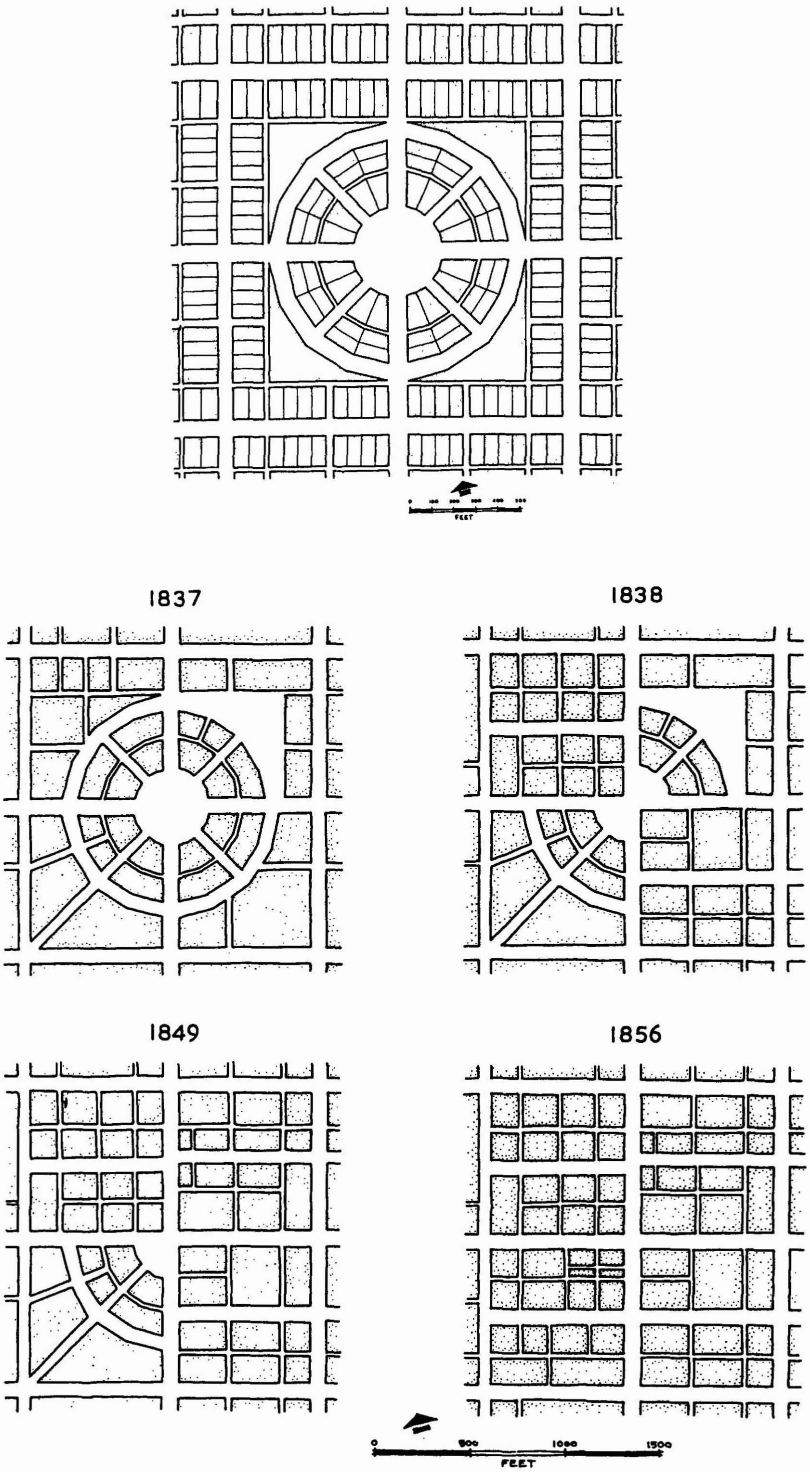


Figure 1.7. The squaring of Circleville 1810–1856 (from Reps, 1965).

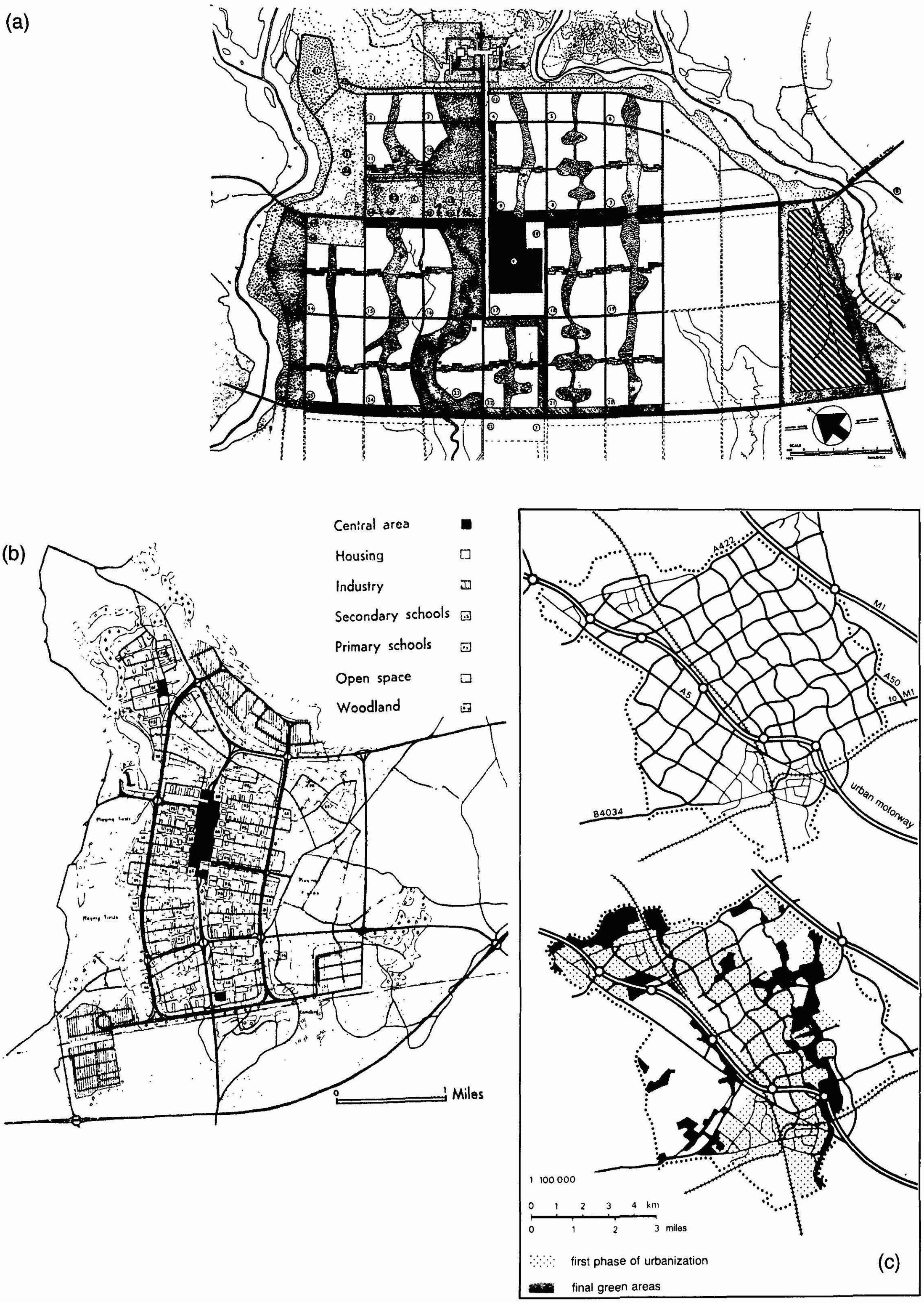


Figure 1.8. The large scale twentieth century grid: (a) Chandigargh (from Kostof, 1991); (b) Hook New Town (from Keeble, 1959); (c) Milton Keynes (from Benevolo, 1980).

its conception is much greater than anything we have illustrated so far in this chapter but its form is still rooted very firmly within conventional Euclidean geometry. However, the plan like so many in the 20th century is more comprehensive, emphasizing strict segregation of uses, as well as a separate landscape – a green grid – which is the complement of the urban transportation and neighborhood grid. Similar types of grid can be seen in some of the British New Towns which we also illustrate in Figures 1.8(b) and (c).

In contrast, Frank Lloyd Wright's *Broadacre City* is a city of low density which sacrifices the rigor of communal tower block living to a more individualist, American style, although he too casts his ideas into a rigid grid. Many other geometric schemes have been suggested since the beginning of the industrial era based on exploiting single principles of urban development: transport around which the linear city such as Sonia y Mata's *Cuidad Lineal* was fashioned in a proposal in 1882 and its application to existing city forms as in the MARS plan for London in the early 1940s, integrated service provision as in Frank Lloyd Wright's later and somewhat extreme reaction to his own *Broadacre City* through the idea of mile high residential superblocks, and in Dantzig and Saaty's (1973) *Compact City* in which all services are concentrated in a city of five or so levels, but built entirely in purely geometric and organized fashion as a machine for living. We illustrate these conceptions in Figure 1.9 where it is now clear that the emphasis has changed a little. The geometry of the ideal town has been relaxed slightly during the 20th century; it is more curvilinear, but still linear nonetheless. It is more organized around new transportation technologies and it is more concerned with land uses and activities than with specific building shapes. However, these ideals are still largely visual in organization and intent, and rarely portray any sense of urban evolution which is so important to the development of cities. We will, however, shift our focus, still concentrating on the visual form of cities in two not three dimensions, but now examining cities which are not dominated by pure geometry, those for which their development is often assumed to be more 'natural'.

1.5 The Organic City

Organic cities do not display obvious signs that their geometry has been planned in the large, although they may well be a product of many detailed and individual decisions which have been coordinated in the small. Therefore it is probably more a hindrance than a help to think of organic cities as being 'unplanned' in contrast to those that have been 'planned', as this represents only the most superficial of reactions to urban form. Thus we will avoid any association between 'organic' cities and the notions of uncoordinated or uncontrolled growth, although we will follow at least the spirit if not the word of Kostof (1991) who characterizes the organic city as:

> ... 'chance-grown', 'generated' (as against 'imposed'), or, to underline one of the evident determinants of its pattern, 'geomorphic'. It is presumed to develop without the benefit of designers, subject to no master plan but the passage of time, the lay

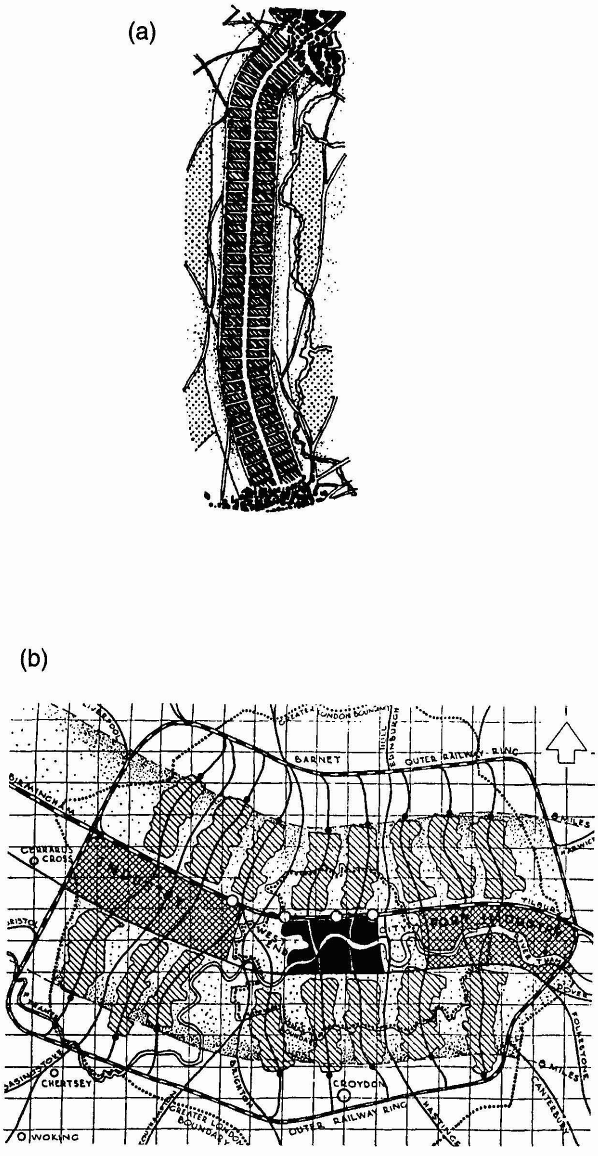


Figure 1.9. Experimental urban geometries: (a) Cuidad Lineal (from Keeble, 1959); (b) The MARS Plan for London (from Keeble, 1959);

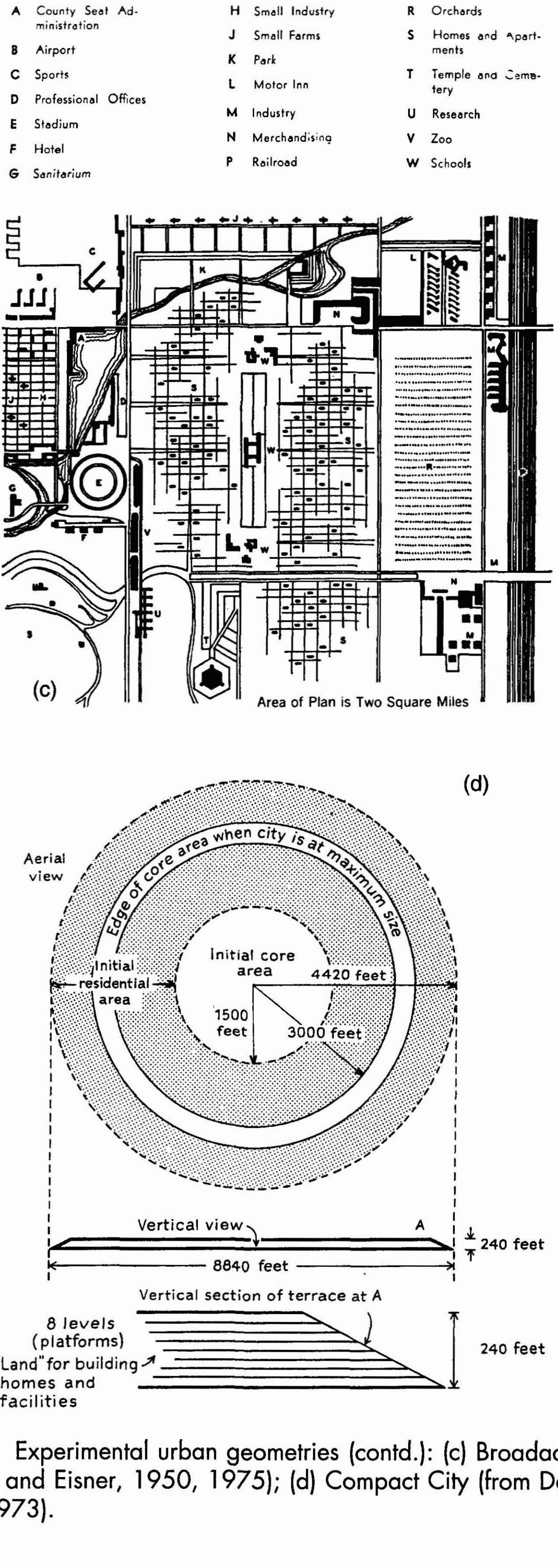


Figure 1.9. Experimental urban geometries (contd.): (c) Broadacre City (from Gallion and Eisner, 1950, 1975); (d) Compact City (from Dantzig and Saaty, 1973).

> of the land, and the daily life of the citizens. The resultant form is irregular, non-geometric, 'organic', with an incidence of crooked and curved streets and randomly defined open spaces. To stress process over time in the making of such city-forms, one speaks of 'unplanned evolution' or 'instinctive growth'. (Kostof, 1991.)

The biological metaphor in city planning has been used since the 16th century, from a time when man first began to move beyond myth to a scientific study of the body and when analogy became one of the dominant ways of making progress in the sciences (Steadman, 1979). In one sense, the idea of the organic city follows this metaphor, especially reinforced in the notion that the organic cities adapt to individual social and economic preferences, to the constraints of the natural landscape, and to the dominant technology of the city. The metaphor has been exploited as cities have grown exponentially in population especially over the last 300 years, and as activities have begun to restructure themselves more quickly through decentralization of functions and increasing locational specialization. The idea of the city as being composed of a 'heart' – the central business district (CBD), of 'arteries' and 'veins' in terms of the hierarchy of transport and communications routes, of 'lungs' in terms of green space and so on, has been writ large across the face of urban analysis and city planning over the last 100 years.

Yet there has been a subtle and growing contradiction between this metaphor and the dominant practice of city planning: although the metaphor has been widely embraced, it has been used both to argue that cities are sometimes poorly-adapted, sometimes well-adapted – that cities should be planned according to the metaphor or against it – that the evolution of cities shows good fit with their requirements or not, and that cities show stability or are pathological in their evolution, exhibiting more cancerous than balanced growth. There is little consistency between these points of view and the preoccupation with the design of cities in visual terms. In fact, in this book, although we will not exploit the terminology of the biological metaphor, much of what we will argue is entirely consistent with it in seeking an understanding of the city which is deeper and less superficially visual than that associated with the traditional geometric model. It is in this sense too, that our interpretation of the city is tilted more towards the organic than the city of pure geometry.

Clearly, cities display a mixture of these two styles, although for over 95% of those which exist and have existed, their form would be seen as being more organic than purely geometric. This is in stark contrast to those cities which are illustrated as examples in the education of city planners where the dominant model is the geometric, cities planned in the large. In fact, examples of cities which developed organically up until the middle ages are conspicuously absent from the historical, certainly the visual record. This may be due to the small size of towns in ancient times, but it is also due to the way towns were represented visually and perhaps of the particular biases which the ancients had toward urban form. However, in modern times, the bias has changed in that towns are now subject to very different levels of organizational control, and building and transport technology than before. Thus there are elements of 'conscious' planning on at least one level in every town, although little or no evidence of planning at higher scales where the focus is upon the growth of entire towns and cities in their urban region.

As we have implied, there is a major problem of representation when we come to examine the visual history of town form. Not only has the type of artistic representation changed as we can clearly see when we examine the forms shown in the figures in this chapter, but the scale at which towns are depicted has altered as they have grown through time, with many more scales of possible representation now than at earlier periods of history. Moreover, the focus of representation has changed. Town plans and maps now are clearly geared to more functional purposes than they were centuries ago and there are a greater variety of possible types of urban map. There are also differences between plans which are designed to show geometric form which often embody yet-to-be-realized ideas in contrast to existing plans which are part of the historical record and rarely designed to show such ideal conceptions.

The biggest differences, however, between urban forms through history are due to size and scale. Before the modern age, most cities were small and compact, with higher densities, much smaller space standards than now, often by orders of magnitude (witness the barracks space in a typical Roman fort), and transport technologies which were much more limited in terms of their ability to move people and goods as well as in their access by ordinary citizens. By far the biggest city before the 17th century was the Rome of the later Empire which at its peak had over one million inhabitants. But Rome was an anachronism. The size of the city was a symptom of the malaise of the Empire for the technology of the civilization was simply unable to sustain such a system. By contrast, the Greek city states rarely grew to larger than 20,000 with only Athens and Syracuse growing to 50,000. Thus before the modern age, urban form was dramatically constrained in contrast to the physical urbanization of the last 200 years. In this view then, the conception of what we might call organic is not independent of either history or culture.

In terms of our present-day notion of organic growth, we only begin to see such forms in the city during the middle ages, and even then the kinds of explosive growth which characterize present-day cities only began in the early 19th century. It has almost been as though there is now 'too much' to plan in contrast to the past, that economic growth and scientific change have reached a threshold in terms of urban growth, beyond which the organic analogy only applies. However, it is more likely that our conception of growth has changed. In history, organic form is associated with slow growth, akin to the gradual accretion of cells, their gradual replacement and renewal which are much closer packed than the way similar units of development are added to and deleted from cities today. Cities now are clearly more dispersed, the use of land is across a much wider range of functions and our concept of irregularity which is embodied in the differences between slow and faster cell growth, is also different. These distinctions are also reflected in the range of urban forms present today in that some cultures where social and economic norms are closer to those of the past than in the west still generate cities which are organic in the older, slower growing sense. For example, the cities of Islam still contain elements of town form which are unaffected by modern technologies and social organization, although such elements rarely exist now in isolation from more modern forms of town. In short, what this implies is that the theories and models which we advocate here are restricted very much to western cultures whose cities are still largely industrial in structure, although rapidly changing to the post-industrial. We do, however, consider that the principles of fractal geometry which we will be developing here are relevant to cities of any time and any culture, but the examples which we have chosen and the scales at which these are depicted are very much rooted in contemporary patterns of urban growth in the west at the city and regional level.

Another important issue relates to the way the organic and geometric principles of urban form vary with respect to scale. At one scale, the city might appear to be ordered in terms of pure geometry while at another it may appear to have no such planned order and be the product of a multitude of local decisions. The example *par excellence* once again is New York. Manhattan developed organically until the early 19th century when the commissioners of the city laid out the island on a regular grid about which all development then took place. But in the wider urban region, development east through Long Island, on the Jersey shore, into Westchester and Connecticut to the north east was not planned on any form of grid, and at this scale of the city region, the growth looks 'unplanned' and explosive. Zooming out even further to megalopolis – the eastern seaboard (Gottman, 1961) – the organic analogy holds although the region contains much pure geometric planning at the local scale in cities such as Washington, Philadelphia and Baltimore. At this wider scale, there has been less contention about the merits of geometric planning although the 20th century has witnessed many attempts at such large scale urban planning as we demonstrated in the last section and illustrated in Figures 1.8 and 1.9. However, notwithstanding our focus on irregularity and organic growth at the urban scale and above in this book, fractal geometry is still applicable at lower scales, and in Chapters 2, 7 and 10 we will indicate how such geometry might be used at these finer scales.

The basic organic model involves the growth of a town from some center of initial growth or seed, the growth proceeding in compact form around the center in waves of development like the rings of a tree. This growth, however, is likely to be distorted by radial lines of transportation along which growth often proceeds faster due to increased access to the center, the ultimate form of town thus resembling some star-like shape. In fact, this model presumes that growth is not constrained by the need for some defensive wall, and until the middle ages and even beyond, such walls tended to minimize distortions forced by the radial and nodal structure of the town in its region. Although there may not have been any overall geometric plan to such early towns, their small size and the intensity of use and density of development must have led to considerable coordination and control of development in social and economic terms which would have had an impact physically. This model represents an abstraction from real growth, but it has become the basis of the organic metaphor: a clear example of the growth and form produced are illustrated in Figure 1.10, taken from Doxiadis's (1968) *Ekistics* which still represents one of the most complete statements of the organic approach to city planning.

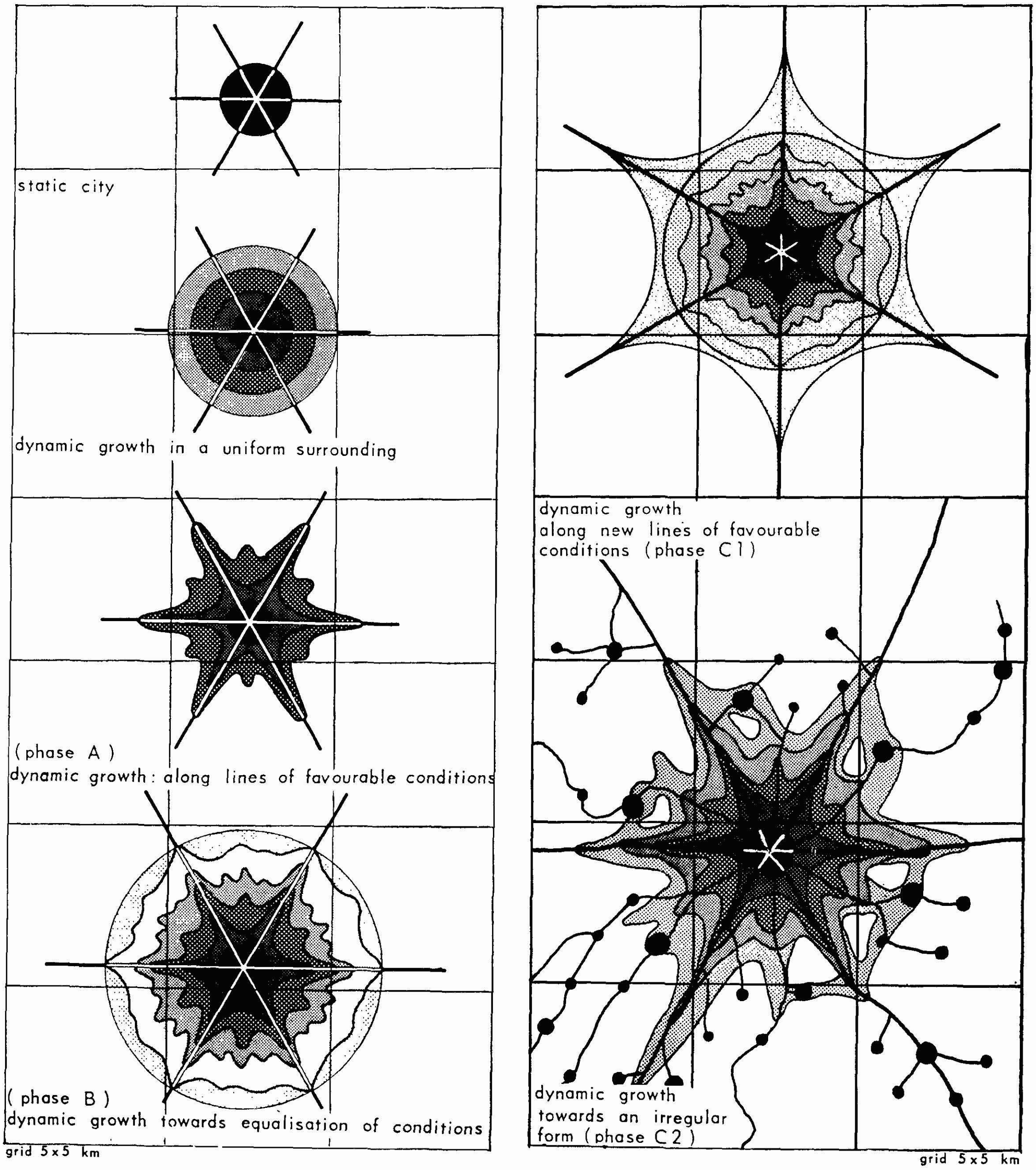


Figure 1.10. The shape of the organically growing city (from Doxiadis, 1968).

The earliest examples of towns in Sumeria demonstrated such slow organic growth in that cells of development composing the town were added incrementally but were highly clustered and continually adapted to the dictates of the physical site. For example, the town of Catal Huyuk (Benevolo, 1980) dating from 6500 BC was built as an accumulation of residential houses all attached to one another but successively adapted in both two- and three-dimensional space across different levels. Such towns are still characteristic of those today in the Middle East and parts of Africa; they were inward looking, based almost entirely on pedestrian traffic, with some sense of regularity in terms of the use of straight lines to demarcate property, but closely hugging the landscape and with no sense of overall visual unity in plan form. In so far as the visual quality of such organic development can be applauded, it is in terms of its informality, its idiosyncracies and its picturesque properties and occasionally in its exploitation of dramatic natural features, but never in terms of the power of its geometry. As we have implied, from these times there are hardly any examples of organic growth which are faster than the century by century adaptation characteristic of these types of town. Where organic growth was faster and larger in scale as in the Athens of Pericles or in second century Rome, the emphasis was not upon transport and decentralization and dispersion of functions, but on adapting the site to the most cultured art and architecture in an effort to mesh the pure geometry of building with the natural geometry of the landscape.

By the middle ages, the slow accretion and adaptation of development to its site so characteristic of the medieval town was well established and there are several illustrative examples. We have already seen the evolution of Regensburg in Figure 1.3 from its Roman grid in the fourth century to its replacement by a huddled mass of different sized and shaped buildings by the year 1100. The development is characteristic of the high feudalism which made its mark on the medieval market town too. A more picturesque example taken from southern Bohemia around 1300 is the town of Cesky Krumlov (Morris, 1979) which shows how the meander of the river, the topography of the river valley in which it sits and the circuitous nature of the transport routes have molded its urban form. There is no sense of the grid or the circle in this type of town, although had this been the New World of the 19th century, a grid would certainly have been imposed with interesting consequences. The town and its medieval development are shown in Figure 1.11. Finally, the medieval town also represents the last example of very slow organic growth where towns were compact and constrained behind their walls in a period of comparative stability when population and economic growth was modest but slow.

By the 17th century, Europe and the Americas were on their way to the industrial era where better transport systems and building technologies were to ultimately lead to much bigger and much lower density cities. The city wall went first, thus enabling the town to begin to conform to its classic star-like shape. In Figure 1.12(a), we show the form of Boston in 1640 (from Reps, 1965), which is reminiscent of the medieval English village, although showing clear evidence of the extent to which the cluster of buildings and space is no longer necessary. This is as good an example of the embryonic radially- concentric modern city as any we can portray. Its form is reinforced by another 80 years of growth in John Bonner's 1722 map (Vance, 1990) shown in Figure 1.12(b), but this illustrates the way in which the 'pictorial' map provides a somewhat less clear way of presenting the salient characteristics of form – the radially concentric nature of the transport pattern and the disposition of development. Nevertheless, like Cesky Krumlov, Boston shows no sense of pure geometry in its plan, but it does show form closely adapted to its physical site.

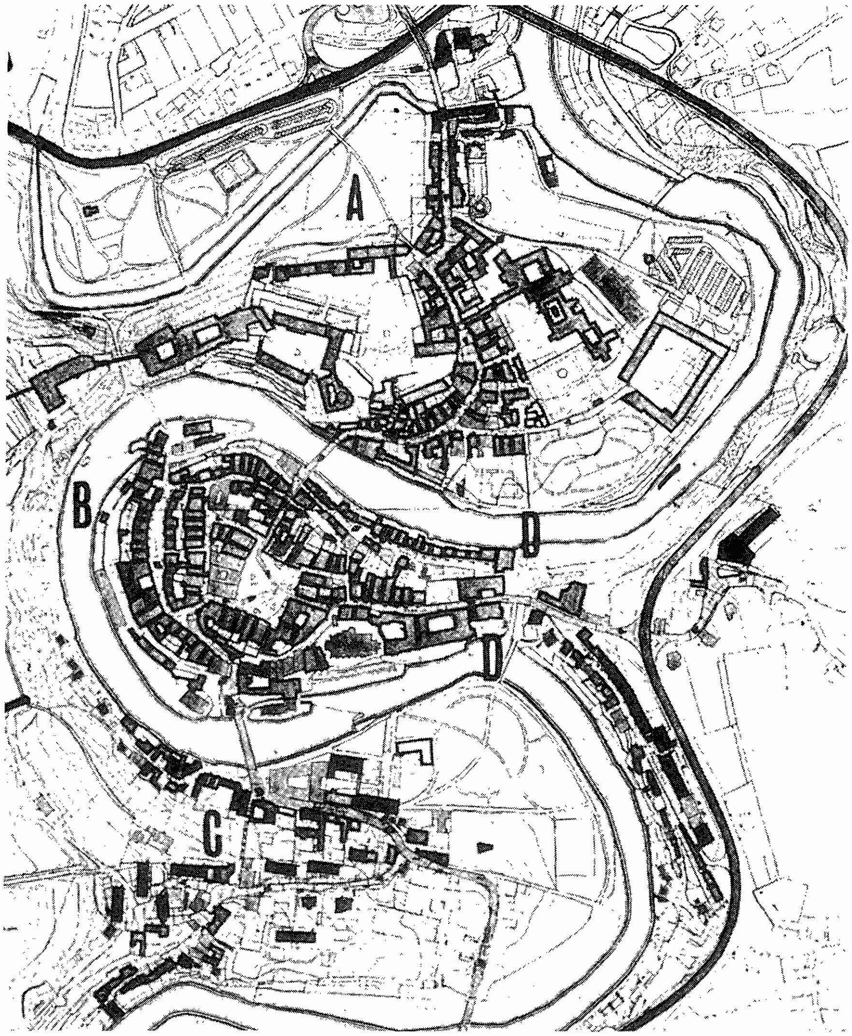


Figure 1.11. Medieval organic growth in Cesky Krumlov (from Morris, 1979).

Although we are unable to present a clear map of the development of Rome in the late Empire, an excellent illustration of urban development in the early 19th century is Clarke's 1832 map which is given in Figure 1.13. Many of the features of organic and geometric form which we have noted in these last two sections are illustrated here; amongst these, are the clear radial structure of the city in its straight roads focussing upon the Palatine Hill, the slow cell-like growth of Rome itself and its medieval development, the distinct Roman monumental architecture of the Coliseum, forum, stadium and so on, and the geometric planning of Pope Sixtus V in the late Renaissance. The wall is still intact to a degree, but the city is spreading across the Tiber and outside its wall, much more characteristic of the present century than earlier ones. There is substantial evidence of the dual mix of traditions of city building in this map, but with an emphasis already on

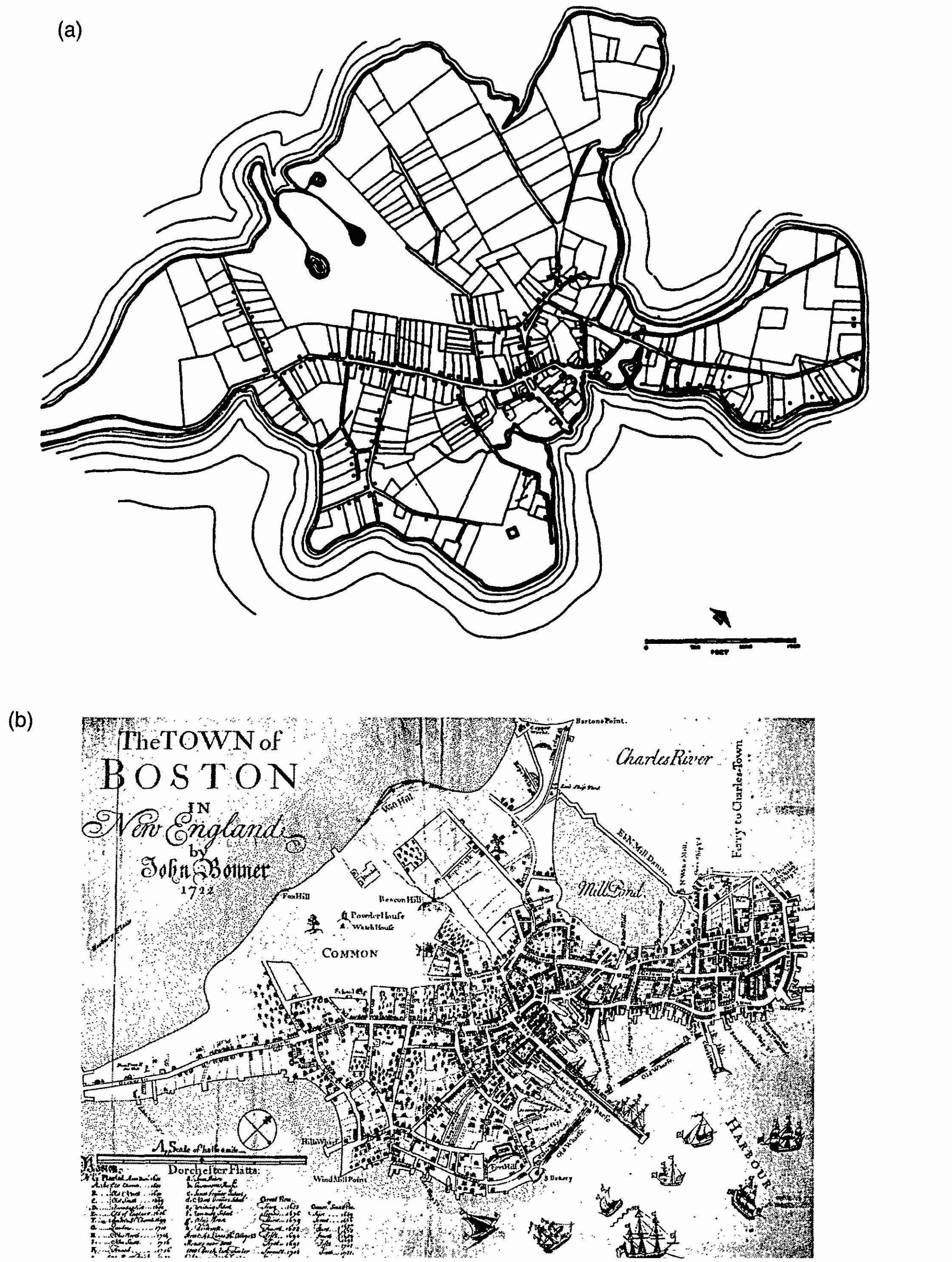


Figure 1.12. Seventeenth century Boston: (a) in 1640 (from Reps, 1965); (b) John Bonner's 1722 map (from Vance, 1990).

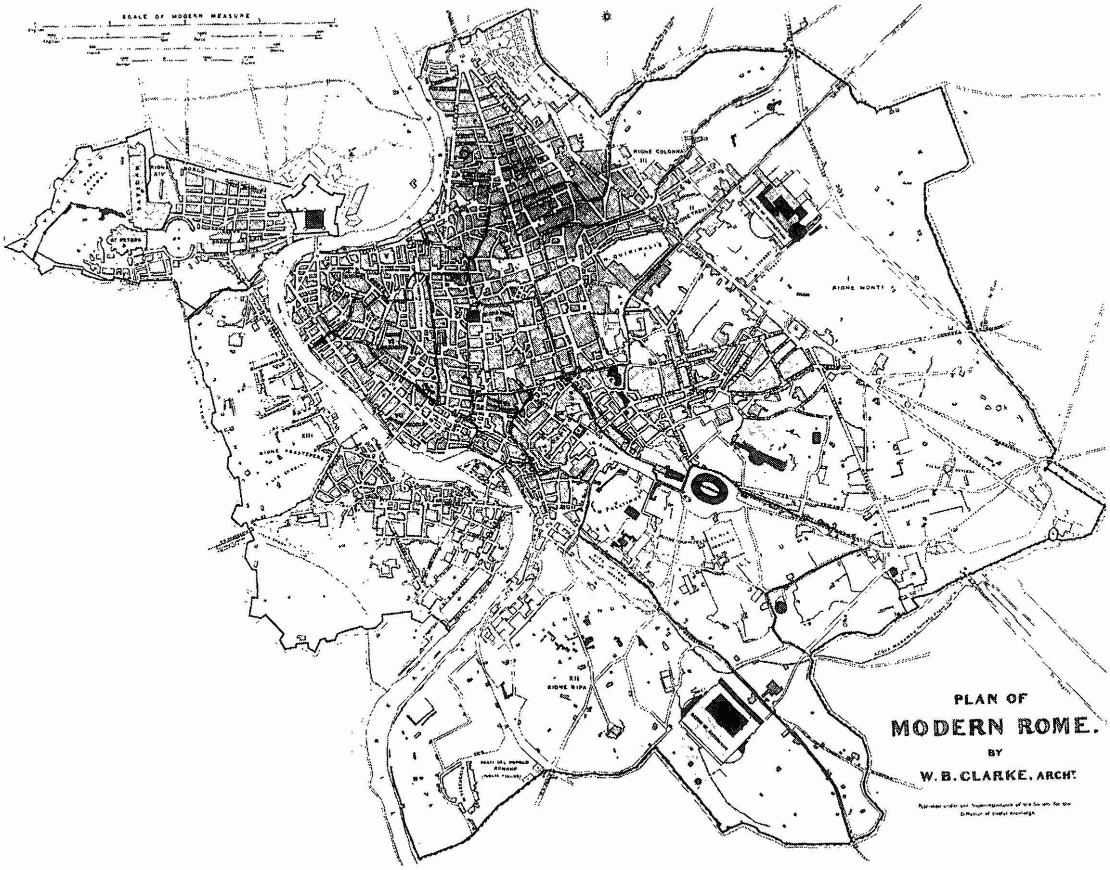
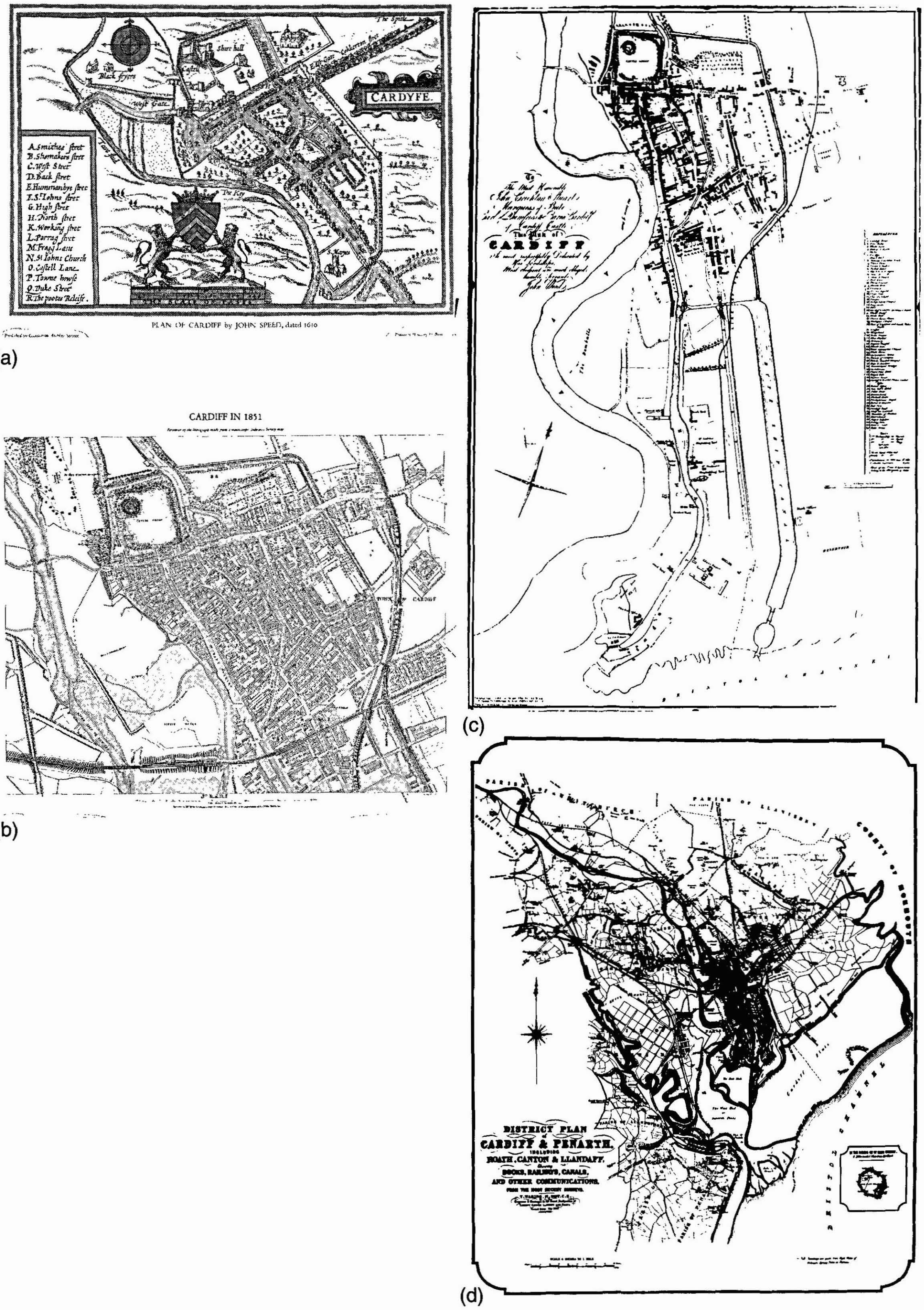


Figure 1.13. The organic growth of Rome by the early19th century (from Morris, 1979).

the power of individual decisions concerning development in contrast to earlier and grander geometric plans for coordination.

Only at this point are we in a position to examine the kinds of urban form which will determine the essence of our explanations in this book. As we have indicated, we do not have a clear and unambiguous time series of urban development in terms of visual (or for that matter any other) form except from old maps, and these are never consistent from time period to time period. However, to give some sense of evolution of urban form over the last 200 years, we show in Figure 1.14 a series of maps for the town of Cardiff from the mid-18th century to the modern day, all reproduced at different scales, showing the way urban development has been depicted differently over this period and also the type of irregularity of form which is the norm rather than the exception in terms of the modern city. We have not abstracted from these maps because we do so in later chapters where we use Cardiff extensively as one of our examples. In fact, in Plate 5.1 (see color section), we show the growth of the city from the late 1880s to 1949 in four stages taken from the relevant maps within the series given in Figure 1.14.

By the mid-20th century, the notion of examining urban form at a larger



(a)

(b)

(c)

(d)

Figure 1.14. The growth of Cardiff, Wales, from the mid-18th century (from the National Museum of Wales).

scale, that of the city and its region, had been well established. In Figure 1.15, we show a diagram from the first edition (1950) of Gallion and Eisner's (1975) book *The Urban Pattern* which is entitled 'The Exploding Metropolis', where the caption implying that such growth is disordered, hence undesirable, both illustrates the predominant concerns of the urban analyst and the ideologies of the city planner. These are the kinds of patterns which we will begin to measure and explain from the next chapter on in our quest to convince that this type of form reveals a degree of order which is considerably deeper than the superficial order associated with the city of pure geometry. Moreover, although we will not dwell very much in this book upon the way in which cities grow and merge forming larger metropolitan areas and urban regions, conurbations in Geddes' (1915, 1949) terms, our analysis and ideas will be entirely consistent with these examples.

The reader is encouraged to skim the figures in later chapters to get some idea of the kinds of forms we will be investigating here, although we will conclude this section with what we consider to be an example of the archetypical urban form for which a theory of the fractal city is most appropriate. Figure 1.16 shows five urban clusters without any scale. There are several points to make here. This pattern of urban development shows no evidence of planned growth, it is radially concentric in structure, it shows

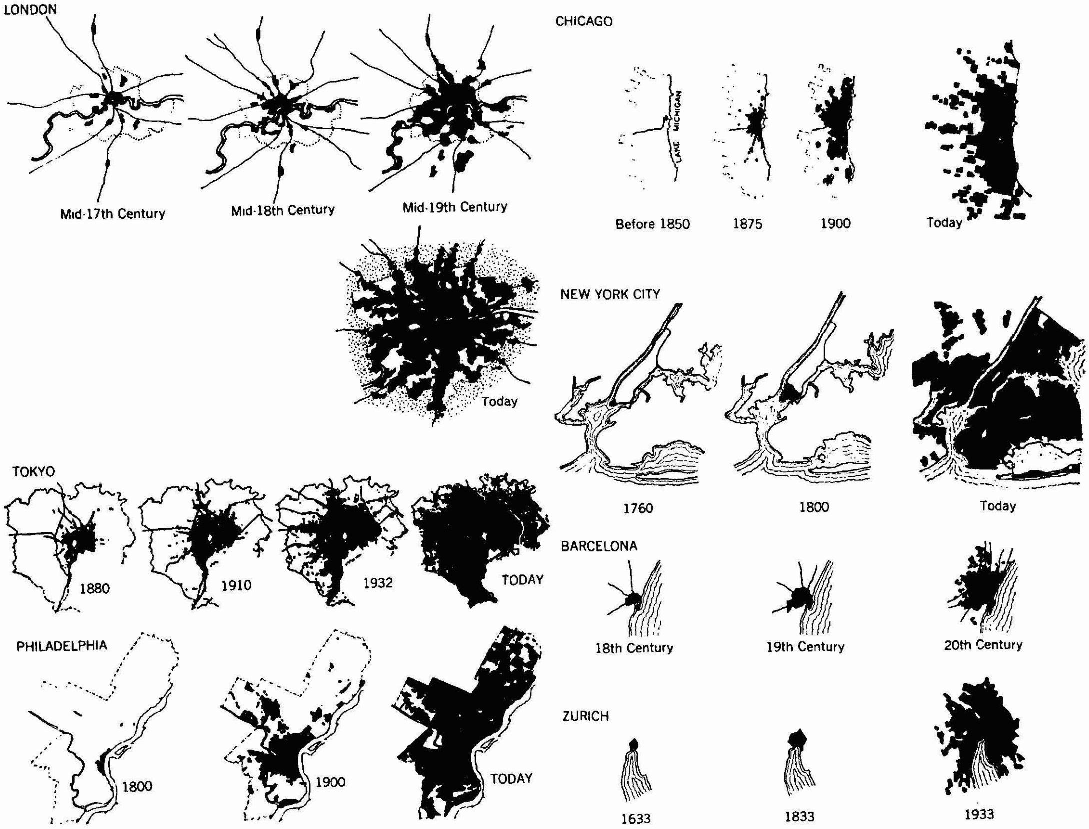


Figure 1.15. The exploding metropolis (from Gallion and Eisner, 1950, 1975).

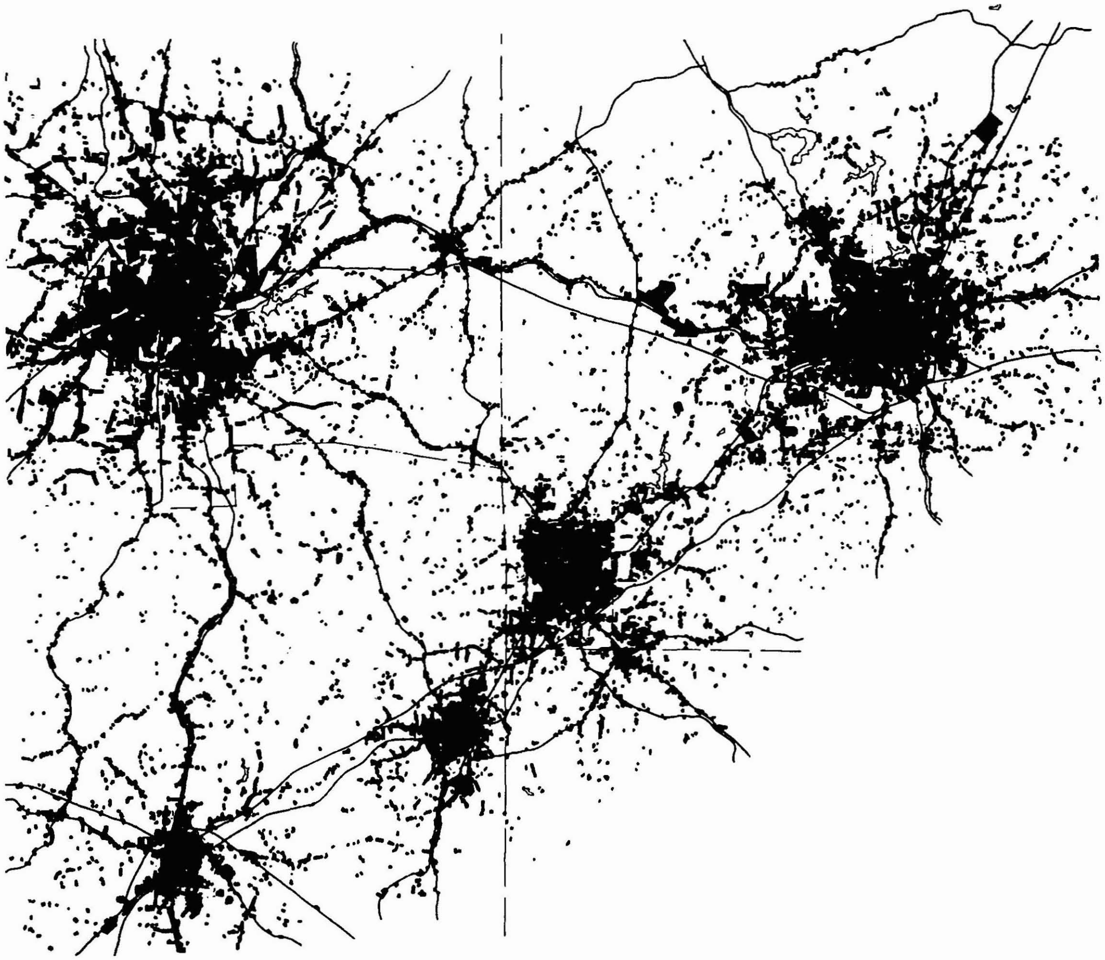


Figure 1.16. Contemporary urban growth patterns (from Chapin and Weiss, 1962).

clusters growing together and it could be at any scale from that of an urban region the size of Tokyo to a cluster of villages and market towns in rural England such as those, for example, we illustrate in Chapter 10. In fact, it is the pattern of urban development in 1958 for five towns in North Carolina – Winston-Salem, Greensboro, High Point, Thomasville and Lexington – taken from Chapin and Weiss (1962). The scale is in fact about 50 miles in the horizontal direction but it could be much larger. This clearly indicates that although our focus is, to an extent, scale dependent, that is, emphasizing urban growth at the city scale up, the patterns we are concerned with do have a degree of scale independence, and our analysis is not restricted to a narrow range which limits the use of our theories and techniques. This whole question of scale will be exceedingly important to our subsequent analysis, and in the next chapter we explore its implications in considerable detail. But it is important to accept that in the application of geometry to human artifacts such as cities, definitions and approaches are always contingent upon the mode of inquiry, the culture to which the analysis applies, and the time at which the application is made. In this sense then, our ideas about the fractal city are appropriate now, and although the principles are likely to be enduring, the examples will change as the approach develops.

1.6 Morphology: Growth and Form, Form and Function

We have made use of the word 'form' extensively already without attempting any definition, for in one sense, the term is self-evident: as D'Arcy Thompson (1917, 1961) implies, form means shape, and in this context, shape pertains to the way cities can be observed and understood in terms of their spatial pattern. In fact, we will need to reflect a little more deeply on the word because our usage here implies a certain approach to geometry and space as well as process and function. Whyte (1968) sums this up when he says: "The word 'form' has many meanings, such as shape, configuration, structure, pattern, organization, and system of relations. We are here interested in these properties only in so far as they are clearly set in space", and this is the usage we will follow here. Form is broader than shape *per se*, although our immediate and first attack on its measurement and understanding is through the notion of shape, in "the outward appearance of things" (Arnheim, 1968). In terms of the study of cities, form will represent the spatial pattern of elements composing the city in terms of its networks, buildings, spaces, defined through its geometry mainly, but not exclusively, in two rather than three dimensions. Yet form can never merely be conceived in terms of these local properties but has a wider significance or gestalt, a more global significance in the way cities grow and change.

The analytic study of form of which this book is a part is always more than it seems at first sight. Form is the resultant of many forces or determinants interacting in a diverse manner through space and time, thus causing the system to evolve in novel and often surprising ways. D'Arcy Thompson (1917, 1961) best sums it up when he says: "In short, the form of an object is a 'diagram of forces'", and in this sense, the study of form without the processes which give rise to it is meaningless. The association of process with form has two clear dimensions. The first is 'growth' which is loosely used in biology and even in city planning to embrace all types of change, and involves the notion that forms evolve through growth, that objects are transformed through the diverse interaction of their forces. This has led to the term 'organic form'. The second dimension relates to function. The various processes which contain the forces which determine form have specific functions and a study of form from the static viewpoint, form at one snapshot in time for example, is often rooted in the quest to understand function. This approach has been widely exploited throughout the arts and sciences, especially in the first half of the 20th century. 'Form Follows Function' has been the battle cry of the Modern Movement in Architecture, although it is somewhat ironic that in its application to city planning by designers such as Corbusier and Wright, the plans produced have rarely followed the motto faithfully, forms being developed which embody the most minimalist, hence the most restrictive of functions. Indeed, it is the task of this chapter and of this book to demonstrate the poverty of urban analysis and city planning which seeks such a rigid interpretation of form.

The term morphology was first coined by Goethe in 1827 as 'the study of unity of type of organic form' (noted in March and Steadman, 1971). Morphology is thus the study of form and process, growth and form, form and function and as Goethe stated: "The formative process is the supreme process, indeed the only one, alike in nature and art" (quoted in Whyte, 1968). Form too is always more than shape, and we will follow Whyte (1968) who speaks of *spatial form* which he defines as comprising external form or visible shape, and internal form which is structure. This brings us back full circle to the idea of form being some manifestation of system with structure being the underlying or invisible form which explains the external urban form, the form which is the subject of our immediate and casual observation. Systems are often studied in terms of their statics or their dynamics, the first implying structure, the second behavior usually in the context of changing structures. Our first grasp of systems, at least those that in some sense are external to us, is in terms of their structure from which we proceed to infer their behavior in the quest to understand their dynamics. In fact, it is system structure of which form is the most superficial characteristic which often provides the basis for classification, the beginnings of scientific study through appropriate description and measurement.

System structures are defined as being composed of elements and relations, the elements being the basic components of the system, the relations defining the way the elements interact and function. Various decompositions of the system into sets of elements define subsystems which it may be possible to associate with, and arrange into, a distinct hierarchy. The various elements, and aggregations thereof into subsystems, may reflect the same form but at different system levels of the hierarchy, and if this conception of organizing the system this way is spatial in any sense, these subassemblies may be *replications of the same form at different scales.* This is an important point for it reflects one of the principles which we will use in the sequel to develop our idea that cities are fractal in form.

There are, however, many ways to describe the elements of the city which usually depend upon the disciplinary perspective of the theory being invoked. Many of these are spatial, although what constitute the key elements will determine whether or not the city system can be subdivided into a strictly spatial hierarchy. For example, the city might be conceived in terms of activity systems of land uses which do not group easily into spatially distinct parts, or in terms of social-organizational groupings which are not obviously spatial in their most significant variations. In fact, many of these systemic descriptions may map only partially onto the strict spatial organization of the city, and thus we consider the approach to be developed here consistent with a variety of related urban theory which is not explicitly spatial. However, the most obvious way to describe cities is in terms of the way they develop. Hamlets become villages, villages towns, towns cities and cities urban regions, all involving a growth and compounding of spatial forces which leave their mark on the evolution of form. The reverse processes of decline are also evident, while in terms of such change, discontinuities and strange cycles can occur, for the evolution is far from remorseless. The basic component or building block of cities is a unit of development, often housing, sometimes called a 'block', smaller than the neighborhood, and these can be usually assembled into a hierarchy of both distinct and overlapping spatial areas. Together with the various communications networks which link these components and all the related functions such as employment, commerce, education, recreation which have their own hierarchies and networks, these compose a complex but rich spatial ordering which manifests itself in a geometry which cannot be captured in the traditions of Euclid. However, from this brief description, it is important to draw out the idea that system structure can be described by relations organized as networks and/or hierarchies, and this will be the path we follow in the rest of this book.

There are many more problems in finding as convenient a representation for the dynamic processes which evolve the city through its functions. Processes are never immediately obvious, or directly observable, and our measurement of them is subject to an uncertainty principle. We do not have time here to speculate on the wide array of theories and methods used to study urban processes which are the subject of inquiry throughout the social sciences. All we can say is that many of the current approaches which at some point enable an understanding and prediction of urban spatial form can be seen as consistent with the ideas we pursue here. For example, the idea of a hierarchy of urban space which results from growth of cities and the development of systems of cities is a basic ordering principle of general systems enabling stable growth and change. Systems, when changed, are changed at the level of their cells rather than more globally, and in this sense, contain a degree of spatial resilience which is manifested in the persistence of their form (Simon, 1969). Moreover, such cellular or local growth by the successive addition or deletion of basic elements also leads to a fitness of the resultant form to its context or environment which can be destroyed through too rapid growth or intervention at an inappropriate level. This is Alexander's (1964) thesis in which he argues that good design or good decision-making in a broader sense must be based on an understanding of the ways the system evolves through the elements within its hierarchy.

Therefore our approach to urban form will be through tracing the 'invisible structure' of relations which underlie the external form or outward appearance of cities, using ideas involving hierarchies and networks and searching for functions which are consistent with the shape of cities and their evolution. We can sketch out such a structure from the top down, illustrating how urban space can be seen as both a hierarchy and a network which in fact represent different sides of the same coin. In Figure 1.17, we show how this is done, beginning with an idealized square geometry, successively subdividing the space in binary terms (1.17(a)), tracing out a perfect and symmetric hierarchy (1.17(b)). The subdivision can also be traced out as a network on the square space as in Figure 1.17(c), and a comparison of (b) and (c) shows that the hierarchy is the network and vice versa. In a simple way the hierarchy might be considered an inverted tree, and the network the same tree in plan form. As we shall see in Chapter 2, such hierarchies provide models of trees and vice versa. We should also note that the system of relations we show in Figure 1.17 is independent of

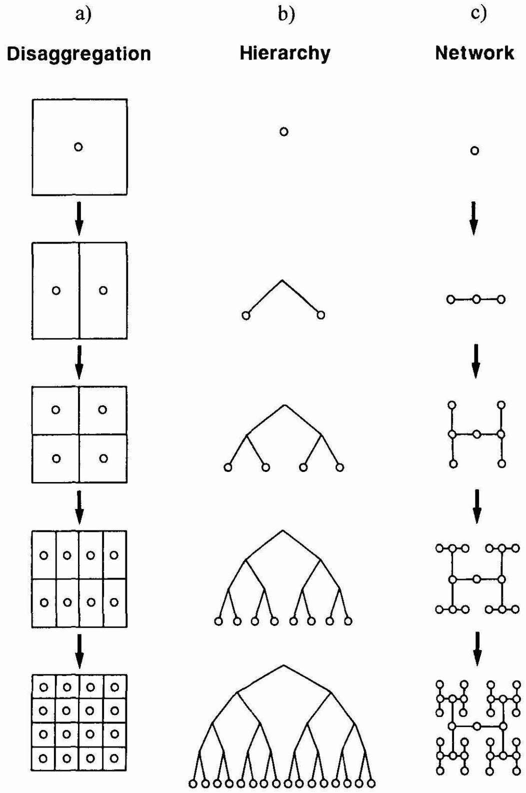


Figure 1.17. Spatial disaggregation: strict subdivision, hierarchy and network structure.

the actual shape of the space to which we apply it: that is, it serves to define either the organic city or the city of pure geometry although as we shall see, it does cast a different light upon the idea of organic form.

If we take a bottom-up approach to the same set of relations, the idea of a distinct hierarchy immediately collapses. As a generating device the hierarchy is efficient, but if we pose the question as to how the elemental units – the most basic grid squares in Figure 1.17 – might aggregate, it is likely that the hierarchy would not capture the degree of diversity within such a structure. If the rule be assumed that each unit aggregates with its nearest neighbor, with the new units overlapping one another in that each element can now belong to one or more aggregate, then what emerges is the semi-lattice structure which we show in Figure 1.18. This is an order of magnitude more complex than the hierarchy; it demonstrates a richness of structure which is in fact still very restrictive in terms of what types of aggregate space might be present in a town, and it is but one of a multitude of possible lattice-like structures. In fact, this is what Alexander (1965) in his famous article 'A City is Not a Tree' suggests is the difference between artificial cities and naturally evolving ones. He says: "What is the inner nature, the ordering principle, which distinguishes the artificial city from the natural city? You will have guessed from my title what I believe this ordering principle to be. I believe that a natural city has the organization

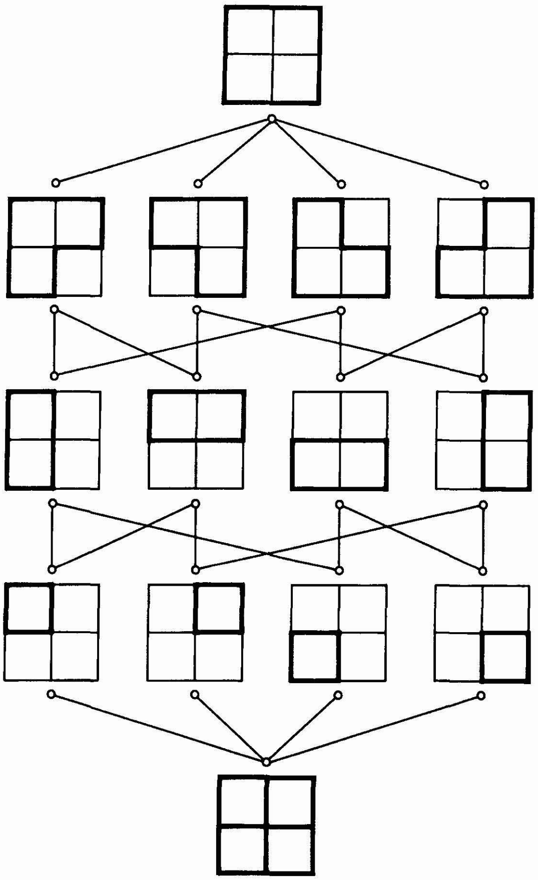


Figure 1.18. Spatial aggregation: overlapping subdivision and lattice structure.

of a semi-lattice; but that when we organize a city artificially, we organize it as a tree".

It is but a short step from Alexander's ideas to the notion that organic cities are not only cities which display an 'irregular' geometry but also cities where that underlying structure of the geometry of its relationships is also 'irregular', or at least asymmetric in the sense of the difference between a lattice and tree. In fact, Alexander (1965) goes on to say that: "... whenever a city is 'thought out' instead of 'grown', it is bound to get a tree-like structure". Thus in terms of Jane Jacobs' (1961) arguments, the doctrine of visual order is doubly at fault for not only imposing a rigid geometry which goes against the natural grain, but also for imposing rigidity of social and functional structure, both of which are highly unrealistic and thus increase rather than diminish the problems they seek to solve. We have raised a theme and an expectation concerning an appropriate geometry of cities which it may appear in subsequent chapters we cannot address or meet. Our succeeding ideas will be dominated by hierarchies and networks, some of which will overlap but most of which will not. However, the complexity we seek to address will take much more than the ideas of this book to report, for this is but a beginning, and the approach we seek to establish is in fact entirely consistent with lattice structures as overlapping hierarchies. The functions we will fit and the patterns we will generate may at first seem strictly hierarchical, but as we will show, this is only the most superficial reaction to the ideas we will introduce. Hierarchies are useful generating principles as we will see in the next section, but the phenomena we seek to explain are always richer, and the models we use always capable of dealing with deeper complexity.

1.7 Urban Hierarchies

Hierarchies, we have argued, are basic organizing devices for describing and measuring the importance of urban functions across many spatial scales. As they are a property of general systems, their import extends beyond individual cities to systems of cities, and thus they present us with the framework for linking local to global and vice versa. In fact, it is the lattice which provides a more appropriate descriptor for this captures the richness of overlap between scales and the somewhat blurred nature of any definition of a distinct and unambiguous scaling. Yet the strict non-overlapping hierarchy which Alexander (1965) rightly ascribes as having been used extensively to purge the natural complexity and variety of cities, is still useful as an initial foray into the way we might organize the relation of scales, one to another, and the fact that we can simplify scales according to a strict hierarchical order does not exclude a richer order from existing within the hierarchy.

Spatial hierarchies relate elements of city systems and systems of cities at successive scales where elements of urban structure are repeated in diverse ways across the range (Berry, 1964). The key idea in this book and the basis of fractal geometry involves identifying systems in which elements are repeated in a *similar* fashion from scale to scale. If this similarity is strong in a geometric sense, then it is referred to as *self-similarity* or in its weaker form as *self-affinity*. We will define these characteristics of the new geometry in Chapter 2 but the idea is all pervasive in the context of cities. In terms of their description, then we will follow a top-down approach in contrast to their generation which always occurs from the ground up. The classic example in the city relates to those routine functions such as retail and commercial services whose frequency and scale of provision is closely tied to the same characteristics of the places where they locate. The largest focus is the CBD, while a loose hierarchy of centers exists throughout the city with lesser numbers of district centers, larger numbers of neighborhood centers, even more local centers and so on, with a size and spacing commensurate with their position in the hierarchy. The same structure exists for the educational and leisure system which is differentiated according to the finer grained differences between functions.

On this basis, cities are usually organized into neighborhoods, typically from 5000 to 10,000 in population, enough to support basic educational and retail functions. Indeed the theory of the ideal city from Plato on has focussed around town sizes which are rarely more than 50,000, often less, implying that a balanced urban structure would be one which contains no more than about 10 neighborhoods (Keeble, 1959). Districts usually comprise two or three neighborhoods, but the differentiation does not end there. Larger towns might comprise smaller, and so on up the hierarchy, while the various transport systems used to enable communication consist of a hierarchy of distributors from primary or trunk down to local, often involving overlapping networks which are further differentiated according to mode (Buchanan *et al.*, 1963).

The smallest examples of urban hierarchies are contained in residential housing layouts where the clearest are those in which vehicle and pedestrian transport is segregated. The layout at Radburn, New Jersey, designed in the late 1920s by Clarence Stein and Henry Wright in the Garden Cities tradition is the prototype (Kostof, 1991). In such layouts where pedestrian routes rarely intersect with vehicular, the networks follow a strict hierarchy and although, in practice, these layouts are generated this way, they are obviously used in a somewhat more flexible fashion. It is in the British New Towns and 1950s housing development that the most archetypical examples can be found. Figure 1.19 provides an example from the town of Coventry where we show the layout simplified as a plan of the road system (1.19(a)), the actual layout of housing (1.19(b)), and the road system as a hierarchy (1.19(c)). Note that Figure 1.19(a) contains an implicit hierarchy of roads where pedestrians can move within the major housing blocks without crossing them, and that these types of layout are reminiscent of many towns in Africa and the Middle East where cul-de-sacs are used extensively to constrain movement.

These layouts are clearly generated artificially, notwithstanding the existence of similar plans which have evolved more naturally. However, various descriptions of cities in terms of the clustering of their neighborhoods and districts also follow strict hierarchies. Abercrombie (1945) in his *Greater London Plan* organized the metropolis into several distinct districts as we show in Figure 1.20, while this idea is also the basis of the development of a hierarchy of small towns, arranged as satellites around an existing central city which is the essence of Howard's (1898, 1965) Garden Cities idea. The kind of geometry which this settlement structure implies is shown in Figure 1.21, and from this there is a clear link to theories of systems of cities which rely upon the notion of a hierarchy of city sizes and hinterlands. Howard's conception of the dependence of small 'new' towns on the central city, at least in the way such settlements were sized and spaced, is clearly consistent with the theory of central places due to Christaller (1933, 1966). By way of conclusion to this section and to systems of relations which we will use in the sequel, we will now show how such theories are consistent with these ideas of hierarchy and city structure at the more local level.

The simplest geometric form of a system of cities is based on an entirely regular grid of basic settlement types – neighborhoods or villages say – which are systematically aggregated into all encompassing regions at successive levels up the hierarchy. We will proceed using this bottom-up approach which is consistent with the way small settlements grow into larger ones, although such systems are often described in the reverse direction. Let us assume a regular landscape of basic urban units which are arranged on a square lattice or grid as in Figure 1.22. In fact, if we assume

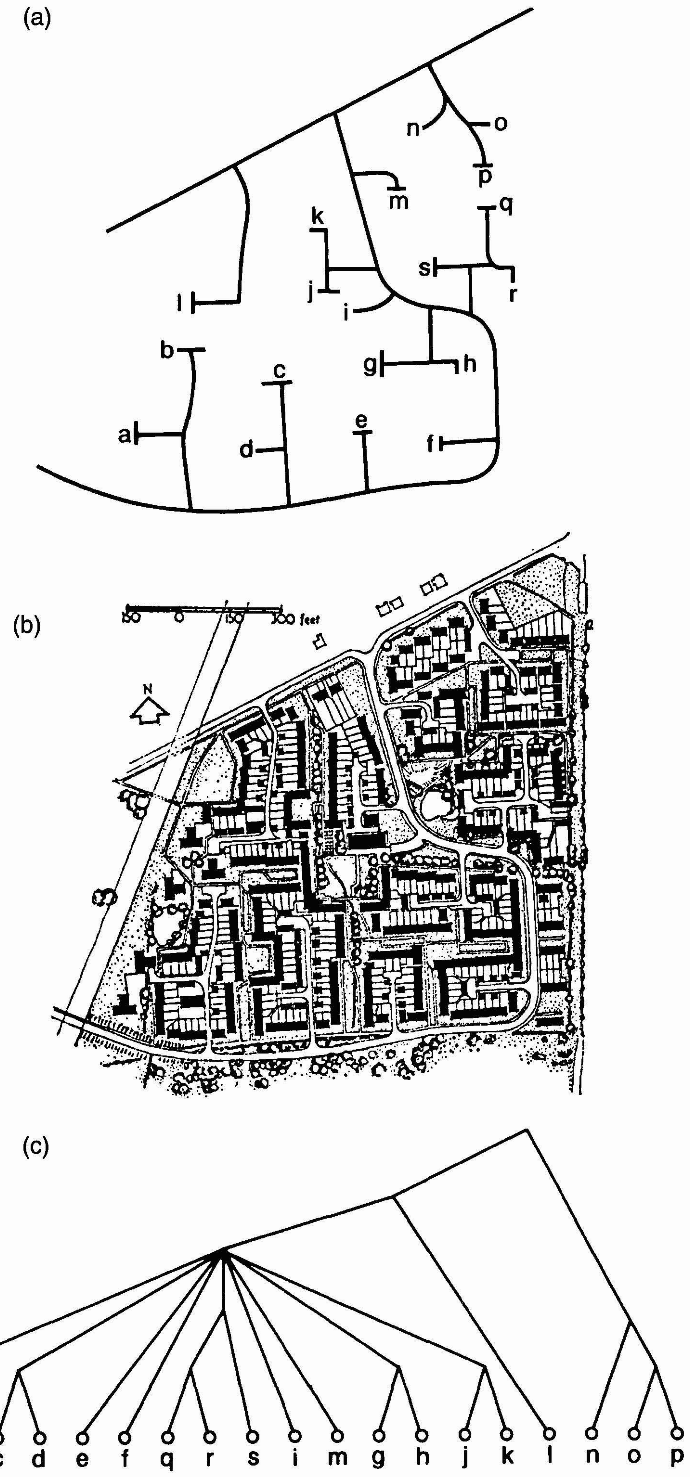


Figure 1.19. Residential layout as hierarchy (after Keeble, 1959).

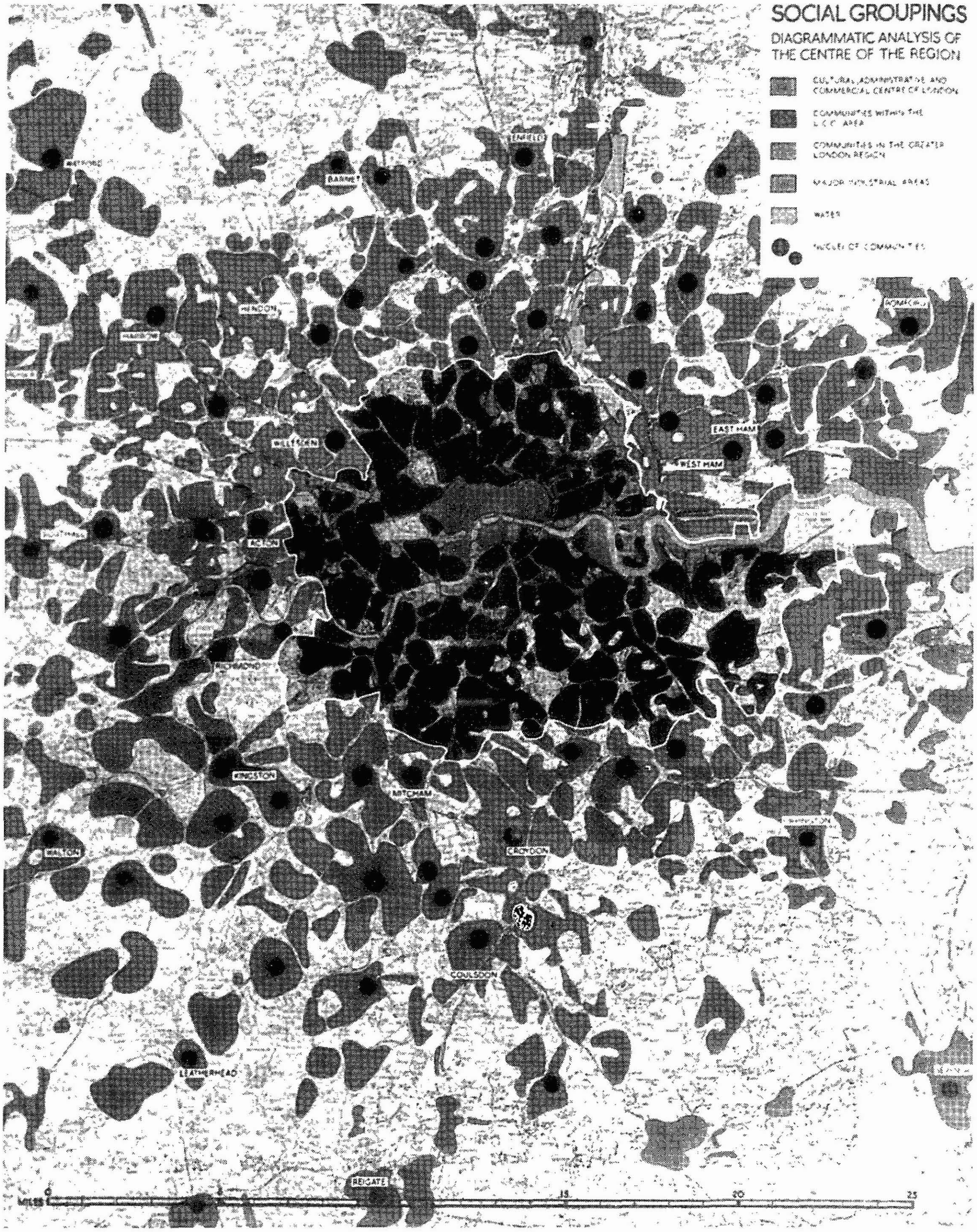


Figure 1.20. The hierarchy of social districts in London (from Abercrombie, 1945).

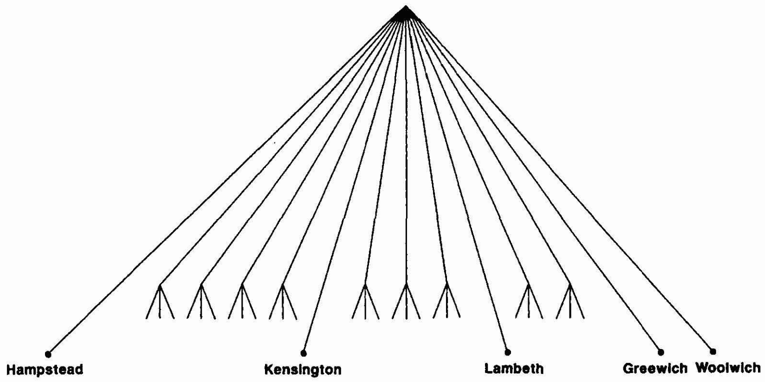


Figure 1.20. Continued.

that these units grow from the smallest seeds, then their areas of influence overlap and the most efficient demarcation between them is clearly the grid; that is, the packing of such hinterlands is most likely to be a grid in this case as shown in Figure 1.22(a). A central place system emerges by defining regularly spaced central places at successive levels of hierarchy, the places in question at each level existing as similar centrally placed locations at all lower levels. The process of aggregating about one major central place in the square grid is shown in Figure 1.22(b).

The number of basic units generated as the aggregation proceeds through levels $i = 0, 1, 2, 3, \ldots$ is given by n^i where n is the number of units in the first aggregate $i = 1$. In the case of the grid in Figure 1.22(b), $n = 9$ (which is formed from an inner grid of 3^2) and thus the sequence of 1, 9, 81, 729, 6561, ... units in typical regions $i = 0, 1, 2, 3, 4, \ldots$ can be formed. The number of units in these regions is given by the recursive relation

$$n^i = n^{i-1}n, \quad i = 0, 1, 2, \ldots, n^0 = 1, \tag{1.1}$$

from which a total population N_i can be calculated in proportion to n^i, where we assume a constant population density ρ. This can be written as

$$N_i = \rho n^i = N_{i-1}n, \quad N_0 = 1. \tag{1.2}$$

From equation (1.2), it is easy to derive another recursive relation relating any earlier aggregation of basic units to a later one, that is smaller aggregates to larger ones. Then noting that $N_{i-1} = N_i/n$

$$N_{i-(j-1)} = \frac{N_i}{n^{(j-1)}}, \quad j = 1, 2, 3, \ldots \tag{1.3}$$

where j is now the 'rank' in the hierarchy with i, the largest index region or the base being associated with the first rank $j = 1$ and so on down the cascade. In essence, equation (1.3) is a rank-size rule of the kind associated with hierarchies based on the Pareto frequency distribution of city sizes (Zipf, 1949). In short, equations (1.1) to (1.3) are power or scaling laws, but with their powers being the ranks or scales themselves. The more usual and simpler rank-size rule is of the form

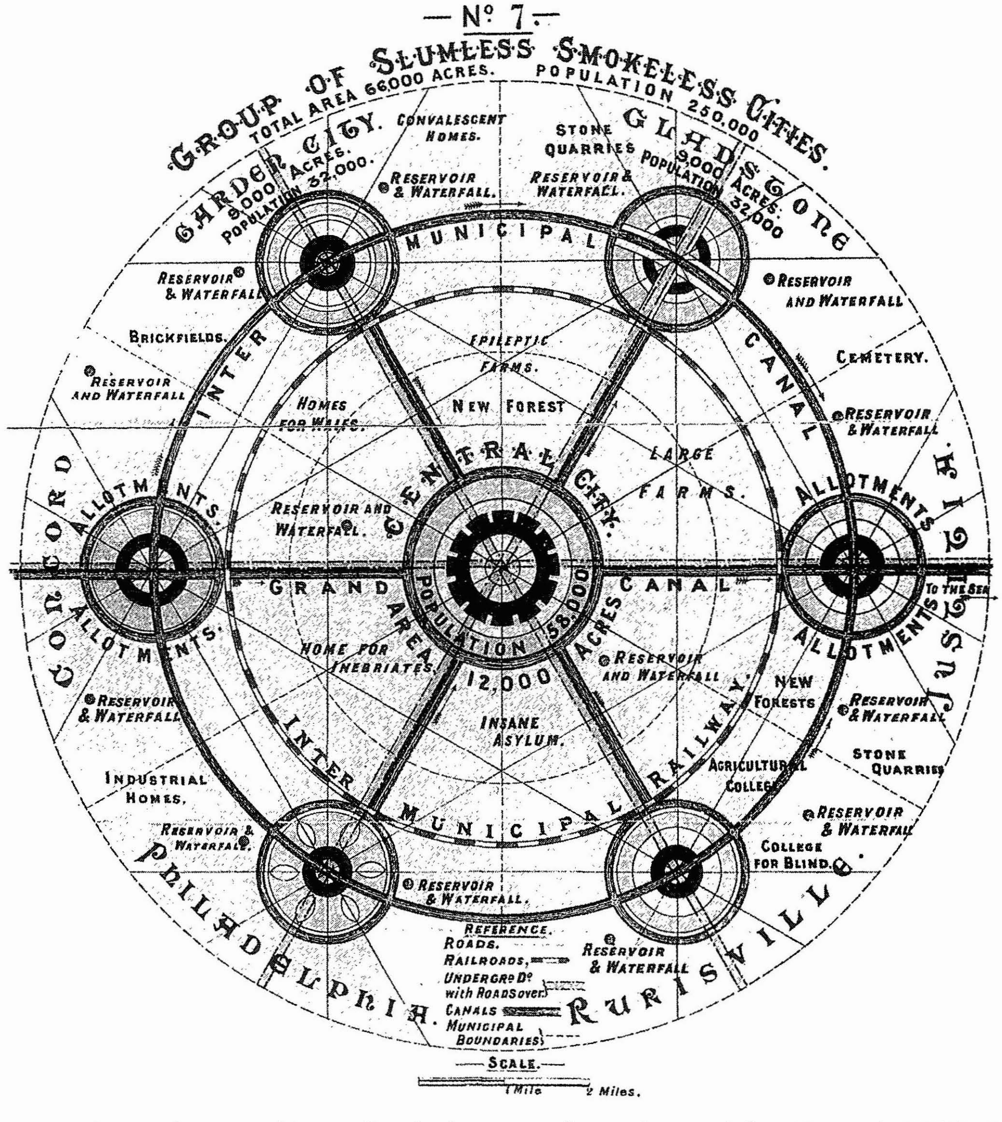


Figure 1.21. Ebenezer Howard's ideal system of social cities (after Howard, 1898, 1965, from Kostof, 1991).

$$N_{i-(j-1)} = \frac{N_i}{j^{\tau}}, \quad j = 1, 2, 3, \ldots \qquad (1.4)$$

where τ is some power usually greater than unity. From equations (1.3) and (1.4), for any level j, $\tau = \{[(j-1)\log(n)]/\log(j)\}$. There are various ways in which equation (1.3) might conform to the simple rank-size rule implied by equation (1.4), most obviously by setting the density of population ρ as some function of the scale or rank. This we will indicate in Chapter 10 where we show how τ might be a function of the fractal dimension D. But for the moment, it is sufficient to note that hierarchies generate power laws and that power laws are one of the bases of fractal geometry.

The hierarchy which is generated in this way is shown in Figure 1.22(c),

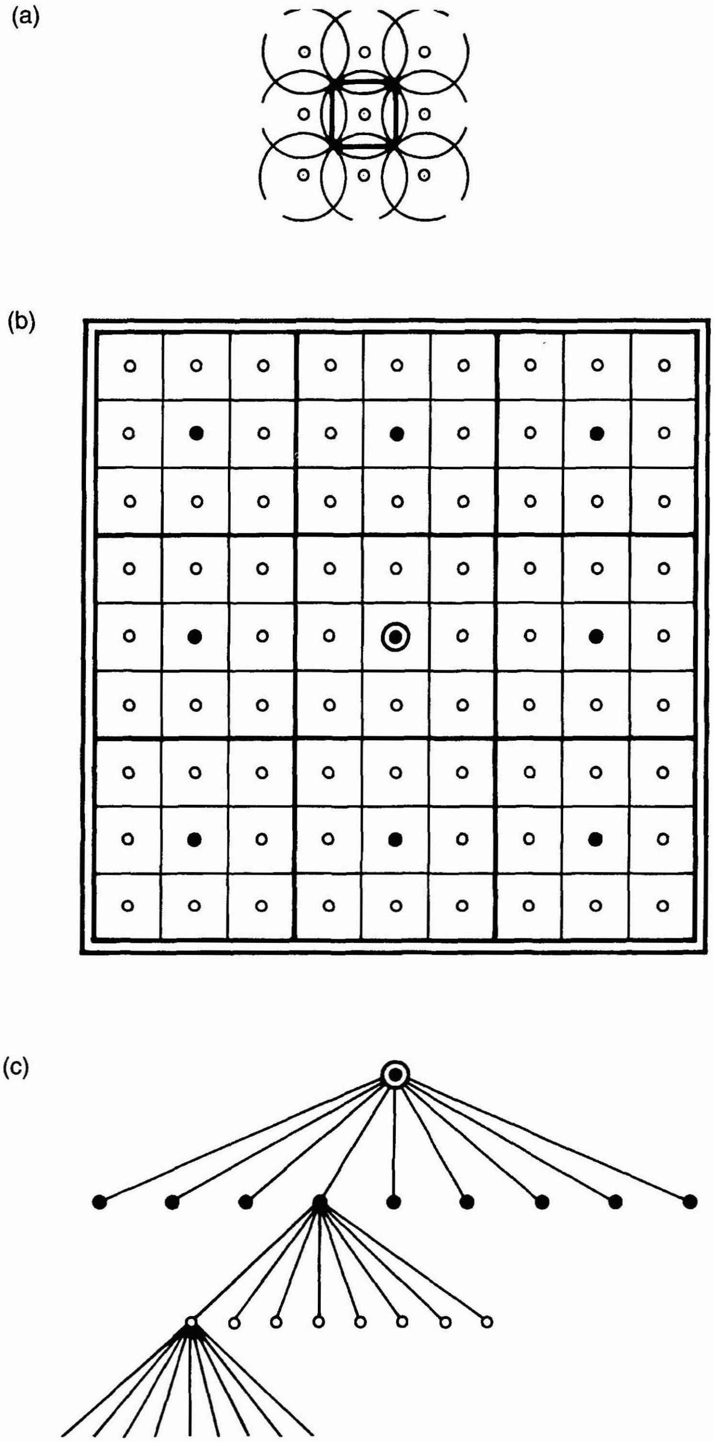


Figure 1.22. Grid geometry and the hierarchy of central places.

and we are immediately drawn to suggesting how this might be made more realistic. If we transform the underlying grid of points to a triangular rather than square net, we generate a packing of basic units which is hexagonal, not square, this now being the geometrical basis of central place theory (Christaller, 1933, 1966). In the square grid system in Figure 1.22, *n* was the basic number of settlement units which was dependent upon a central place at the next order of hierarchy, in that case *n* being equal to 9. The usual approach is to assume that hinterlands defining the dependence of places on a center, share basic settlement units, and in Figure 1.23 we show how

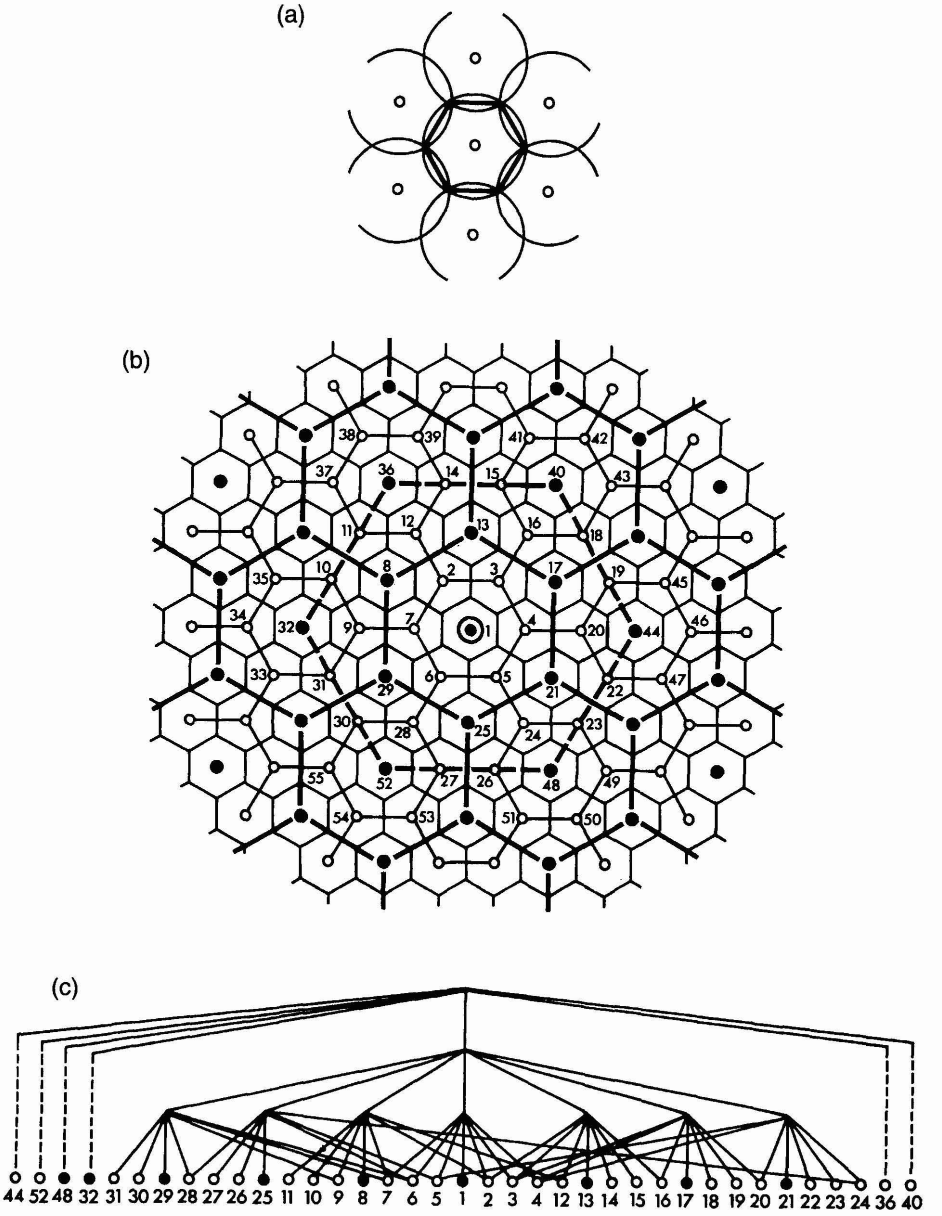


Figure 1.23. Hexagonal geometry and the lattice of central places.

this might be so. Figure 1.23(a) shows the basic hexagonal packing while 1.23(b) shows the way successive hinterlands are constructed with each of six places defining the hexagonal trade area around each center at a given level in the hierarchy, being shared with two other adjacent hinterlands. In this case, the number of dependent settlements within each aggregate at a given level of hierarchy is $n = [1 + (1/3)6] = 3$. Different aggregations which still employ a hexagonal geometry are possible with $n = 4, 7, 9, 12$ and so on, these being called central place systems with given k values, the k defined here as n, the number of dependent settlements between each level (Haggett, Cliff, and Frey, 1977).

In the case of the system shown in Figure 1.23(b), this is based on the most minimal of hexagonal tessellations which, using equation (1.2), generates a sequence of 3, 9, 27, 81, 243, ... basic settlement units at successive levels of hierarchy. Moreover, because of the split dependence of centers on adjacent hinterlands, the hierarchy is, in fact, a lattice of overlapping regions, and this is shown in Figure 1.23(c). It is possible to further increase the realism of these types of systems. By letting the integer n vary at different levels i, and over space, considerable distortions in the central place landscape can be produced (Isard, 1956). The theory is one of the cornerstones of human geography, and although we will show at various points in this book how this structure is consistent with fractal geometry, we have introduced enough to give the reader a flavor of how it might connect to the theories we will espouse here. In fact, the development of central place theory and fractal geometry constitutes a study in its own right, and already a beginning has been made by Arlinghaus (1985). To complete this rapid but long survey of the geometry of cities, we will now focus the conclusions to this chapter on the need for introducing a geometry of the irregular into city systems, noting briefly how these might be linked to other formal approaches to urban design which have emerged over the last 20 years.

1.8 A New Geometry

In this chapter, we have reviewed the study of shape in two ways: first, in terms of the simplest geometry used by those intent on developing the doctrine of visual order, and second, in terms of more abstract geometrical relations, hierarchies and networks, used by those seeking a deeper meaning to spatial order in the city. Whilst Euclidean geometries are largely descriptive and difficult to link to the underlying processes of growth explicitly, the geometry of relations used to show how space and shape within the city is ordered, does begin to suggest ways of unravelling the complexity of urban form. But there have been a succession of approaches to urban form developed over the last 20 years which build on more systematic, mathematical ideas, linking surface to underlying structure and process. We have already noted the coincidence of hierarchical ideas in design pioneered by Alexander (1964), for example, with those in human geography based on central place theory, and it is worth noting that the formality of these ideas has been even further relaxed by Alexander *et al.*

(1977) amongst others in the search for appropriate frameworks for explaining the diversity and richness of urban form. This stream of work will continue to inform the ideas presented in later chapters.

Several formal approaches to shape and layout from the architectural to the city level are built around the ideas of relations or connectivity, a natural starting point being the theory of networks or graphs (March and Steadman, 1971). The idea of a graph as the dual of continuous spatial subdivision has become the basis of architectural morphology in terms of building plans (Steadman, 1983), while more recent work has sought to develop the theory of spatial order using shape grammars built on the basic ideas of mathematical linguistics originally inspired by Chomsky (March and Stiny, 1985). These approaches do not, however, directly broach the notion that form is complex and irregular but ordered, and hence explicable. Perhaps the emphasis within architecture on the ultimate order imposed by Euclidean geometry in building structures has inhibited discussion of irregularity in form which exists at every scale, but only becomes strongly apparent at the larger scales of the city and the metropolis. In this sense, architecture is rooted in the idea of the planned form in contrast to more naturally evolving 'organic' structure, and as Steadman (1979) implies, the biological analogy, although exploited in a casual way, has had less impact on the way designers design. In fact, the development of shape grammars and their linking to cellular automata implies that at the level of buildings, such approaches could well begin to address concepts of irregularity and growth if developments in complexity theory from this perspective gain influence as appears likely at present.

Two other approaches are worth noting. Hillier and Hanson's (1984) approach to spatial form is at a slightly higher scale than the architectural, and they base their ideas on measuring the actual network qualities of neighborhoods and districts up to entire cities. Their approach turns space inside out with a strong emphasis on the way buildings are connected through their external spaces, employing many statistics associated with the patterns of connectivity described using graph theory. Ideas of growth and change are more central to their approach which is clearly based on a concern for the organic in contrast to planned evolution of city systems. Finally, we should note the emerging body of work on treating building and urban systems and their design using cognitive theory, particularly knowledge-based systems which in turn link these ideas back to shape grammars and the morphology of graphs (Coyne *et al.*, 1990).

Yet there is a need for a geometry that grapples directly with the notion that most cities display organic or natural growth, that form cannot be properly described, let alone explained, using Euclidean geometry, that urban form must be related to the underlying theories of the city which form the conventional wisdom of urban economics and human geography. We have implied that such an approach would grapple with the geometry of the irregular but at this point we must also recognize that there are many types of regularity, which do not fit within the traditional Euclidean paradigm, often incorrectly attributed to geometries of the 'irregular'. Such a geometry must deal directly with the notion that our assumptions of continuity when it comes to urban form must be more sophisticated, that shape is not continuous and manifests many discontinuities at the levels of lines and surfaces, but that the way we articulate its dimensions as discrete or discontinuous is too strict an order and must be relaxed to embrace the notion of continuous variation.

This would be a geometry that went beyond the superficial description of form, that built on the essential idea of linking form to function, of form to process, of statics to dynamics, a geometry commensurate with D'Arcy Thompson's (1917, 1961) original quest for a geometry of growth and form. That geometry has emerged during the last 20 years in the geometry of fractions, of shapes that do not display the clean lines and continuity of Euclidean geometry. As Porter and Gleick (1990) in their book *Nature's Chaos* so vividly portray, the geometry which has emerged, fractal geometry, is as much about the artificial world as the natural. They say:

> A painter hoping to represent the choppy ocean surface can hardly settle for a regular array of scalloped brush strokes, but somehow must suggest waves on a multiplicity of scales. A scientist puts aside an unconscious bias toward smooth Euclidean shapes and linear calculations. An urban planner learns that the best cities grow dynamically, not neatly, into complex, jagged, interwoven networks, with different kinds of housing and different kinds of economic uses all jumbled together.

This is a geometry of order on many scales, a geometry of organized complexity which we will begin to develop and apply in the next chapter.